Criterion 1	Curricular Aspects	100

1.1 Curricular Planning and Implementation (20)

1.1.1The Institution ensures effective curriculum planning and delivery through a well-planned and documented process including Academic calendar and conduct of continuous internal Assessment

Table of Contents

SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Approved by AICTE, New Delhi
Affiliated to Anna University, Chennai

Department : CSE
Subject Code & Name : MA3151, Matrices and Calculus.
Class & Batch : I Sec B, 2024-2028.
Semester : I

CONTENTS – COURSE FILE

S.NO	PARTICULARS	REMARKS
1	Time Table	✓
2	Student name list	✓
3	Student arrear list	—
4	Subject Information Record	✓
5	Syllabus	✓
6	Lesson Plan	✓
7	Test Plan for the Subject	✓
8	Result Analysis	✓
9	Corrective Action Report	✓
10	Quality objective monitoring record	✓
11	Internal test mark sheet(Consolidated)	✓
12	Internal test question paper with answer key	✓
13	Model question paper with answer key	✓
14	Slip test question paper with answer key	✓
15	Sample Answer paper for all test(Min-3)	✓
16	Content beyond the syllabus	✓
17	Tutorial Class – schedule and content	✓
18	Assignment – schedule and paper	✓
19	PPT - handout	✓
20	Question bank	—
21	Sample university question papers(min 5 QP-recent exam)	✓
22	Personal Log book – Updated	
23	Lecture Note	
24	Special Class if any, Approval letter, Schedule, content covered.	

	Prepared By	Approved By
Sign:		
Name:	G. Jeevanantham.	M. Sathya
	Faculty	HoD

SASURIE
COLLEGE OF ENGINEERING

CLASS TIME TABLE

Department Science and Humanities

ACADEMIC YEAR : 2024-2025 (ODD) Semester : I

CLASS: I B.E - CSE

HOUR	I	II		III	IV		V	VI		VII	VIII
DAY/ TIME	09.00 a.m. TO 9.55 a.m.	09.55 a.m. TO 10.50 a.m.	10.50 a.m. TO 11.05 a.m.	11.05 a.m. TO 12.00 a.m.	12.00 a.m. TO 12.55 p.m.	12.55 p.m. TO 1.40 p.m.	1.40 p.m. TO 2.20 p.m.	2.20 p.m. TO 3.00 p.m.	3.00 p.m. TO 3.15 p.m.	3.15 p.m. TO 3.55 p.m.	3.55 p.m. TO 4.35 p.m.
I					MAT			MAT			
II				MAT							
III	MAT	MAT	BREAK			LUNCH			BREAK		
IV										MAT	
V				MAT							MAT
VI							MAT				

S.No	Subject Code	Name of the Subject	Abbreviation	Name of the Staff & Dept.	No of hours
1	MA3151	Matrices and Calculus	MAT	G.Jeevanantham AP/MAT	10
CLASS ADVISOR : S.Venkatesan & V. Deepan				TOTAL	10

	Prepared by	Verified by	Authorized by
Sign:			
Name:	Mr.G.Jeevanantham	Mrs.M SATHYA	Dr.M.VIJAYAKUMAR
	Time Table Incharge	HOD	PRINCIPAL

ॐ

SASURIE COLLEGE OF ENGINEERING

SASURIE INSTITUTIONS

Vijayamangalam, Tiruppur.

DEPARTMENT OF SCIENCE AND HUMANITIES

DEPARTMENT : B.E CSE

YEAR / SEM : I / I

ACADEMIC YEAR : 2024-2025

S.No	Reg Number	Name of the Student	Dayscholar/ Hostel
1	732424104062	Mathavan.V	Hostel
2	732424104063	Merlin jenisha Mery.M	Dayscholar
3	732424104064	Methini.L	Dayscholar
4	732424104065	Mohammad Fayaz.S	Dayscholar
5	732424104066	Mohammed Suhail.S	Dayscholar
6	732424104067	Monisha.N.P	Dayscholar
7	732424104069	Nathiya.R	Dayscholar
8	732424104070	Naveenkumar.R	Dayscholar
9	732424104071	Nivetha.P	Dayscholar
10	732424104072	Pradeepa. L	Dayscholar
11	732424104073	Prapeena.B	Dayscholar
12	732424104074	Prema.K	Hostel
13	732424104075	Priyadharsan.V	Dayscholar
14	732424104076	Ragul.O	Hostel
15	732424104077	Ragul.R	Hostel
16	732424104078	Rajapriyan.K	Hostel
17	732424104079	Sabarinathan. K	Dayscholar
18	732424104080	Sabarish.V	Dayscholar
19	732424104081	Sanjay.M	Dayscholar
20	732424104082	Sanjay Kumar.R	Dayscholar
21	732424104083	Sanmathi.S	Dayscholar
22	732424104084	Sanofar. M	Dayscholar
23	732424104085	Santhiya.S.V	Dayscholar
24	732424104086	Sarathi.G	Dayscholar
25	732424104087	Saravanan. M	Dayscholar
26	732424104088	Sarmitha.P.V.K	Dayscholar
27	732424104089	Sasvitha. S	Hostel

28	732424104090	Senthamizh.J	Dayscholar
29	732424104091	Shalika.S	Dayscholar
30	732424104092	shamprasanth.S	Hostel
31	732424104093	Shyam Siddharth.S	Dayscholar
32	732424104094	Sidharsan.M	Dayscholar
33	732424104095	Soundarya.S.T	Dayscholar
34	732424104096	Sowmiya.B	Dayscholar
35	732424104097	Sowmiya.M	Dayscholar
36	732424104098	Sriharishma.R	Hostel
37	732424104099	Sriram.K	Hostel
38	732424104100	Srishanth.K	Hostel
39	732424104101	Stephy Rose Manna.A	Hostel
40	732424104102	Sujith.S	Dayscholar
41	732424104103	Suthishna.S	Dayscholar
42	732424104104	Swetha.V	Hostel
43	732424104105	Tharunkumar.K	Hostel
44	732424104106	Thenmozhi. M	Dayscholar
46	732424104107	Thirumurugan.M	Dayscholar
47	732424104109	Varshini. N	Dayscholar
48	732424104111	Velusamy.M	Dayscholar
49	732424104112	Venkatraj.R	Hostel
50	732424104113	Vidhyavarshini.M	Dayscholar
51	732424104114	Vignesh.M	Hostel
52	732424104115	Vimalesh. R. K	Dayscholar
53	732424104116	Vinthia varshini.S	Hostel
54	732424104117	Vishal.M	Dayscholar
56	732424104118	Yogalakshmi.G	Dayscholar
57	732424104118	Yuvashri.S	Dayscholar

	Prepared by	Verified by
Sign:		
Name:	G. Jeevanantham.	M.Sathya
	Faculty	HOD

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SASURIE
COLLEGE OF ENGINEERING
SASURIE INSTITUTIONS
Vijayamangalam, Tiruppur.

SUBJECT INFORMATION RECORD

Department	:	CSE
Subject	:	Matrices and calculus.
Year	:	I
Semester	:	I
Last year handled by	:	N. Sharanya
Percentage of Result (last year)	:	93%.
Quality Objectives	:	To develop the use of matrix algebra techniques that is needed by Engineers for Practical applications
Reference Book	:	1. Anton, H. Bivens I and Davis S. "calculus" wiley, 10^{th} Edition 2016. 2. Narayanan. S and Manicavachagan pillai "calculus" volume I and volume II. 3. Ramana. B.V. "Higher Engineering mathematics"

	Prepared By	Approved By
Sign:	[signature]	M. [signature]
Name:	G. Jeevanantham	M. Sathya
	Faculty	HOD

MA3151 **MATRICES AND CALCULUS** **L T P C**

3 1 0 4

COURSE OBJECTIVES:

- To develop the use of matrix algebra techniques that are needed by engineers for practical applications.
- To familiarize the students with differential calculus.
- To familiarize the student with functions of several variables. This is needed in many branches of engineering.
- To make the students understand various techniques of integration.
- To acquaint the student with mathematical tools needed in evaluating multiple integrals and their applications.

UNIT I MATRICES 9 + 3

Eigenvalues and Eigenvectors of a real matrix – Characteristic equation – Properties of Eigenvalues and Eigenvectors – Cayley - Hamilton theorem – Diagonalization of matrices by orthogonal transformation – Reduction of a quadratic form to canonical form by orthogonal transformation – Nature of quadratic forms – Applications : Stretching of an elastic membrane.

UNIT II DIFFERENTIAL CALCULUS 9 + 3

Representation of functions - Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules) - Implicit differentiation - Logarithmic differentiation - Applications : Maxima and Minima of functions of one variable.

UNIT III FUNCTIONS OF SEVERAL VARIABLES 9 + 3

Partial differentiation – Homogeneous functions and Euler's theorem – Total derivative – Change of variables – Jacobians – Partial differentiation of implicit functions – Taylor's series for functions of two variables – Applications: Maxima and minima of functions of two variables and Lagrange's method of undetermined multipliers.

UNIT IV INTEGRAL CALCULUS 9 + 3

Definite and Indefinite integrals - Substitution rule - Techniques of Integration: Integration by parts, Trigonometric integrals, Trigonometric substitutions, Integration of rational functions by partial fraction, Integration of irrational functions - Improper integrals - Applications: Hydrostatic force and pressure, moments and centres of mass.

UNIT V MULTIPLE INTEGRALS 9 + 3

Double integrals – Change of order of integration – Double integrals in polar coordinates – Area enclosed by plane curves – Triple integrals – Volume of solids – Change of variables in double and triple integrals – Applications: Moments and centres of mass, moment of inertia.

TOTAL:60PERIODS

COURSE OUTCOMES:

At the end of the course the students will be able to

CO1: Use the matrix algebra methods for solving practical problems.

CO2: Apply differential calculus tools in solving various application problems.

CO3: Able to use differential calculus ideas on several variable functions.

CO4: Apply different methods of integration in solving practical problems.

CO5: Apply multiple integral ideas in solving areas, volumes and other practical problems.

TEXT BOOKS:

1. Kreyszig.E, "Advanced Engineering Mathematics", John Wiley and Sons, 10th Edition, New Delhi, 2016.
2. Grewal.B.S., "Higher Engineering Mathematics", Khanna Publishers, New Delhi, 44th Edition , 2018.
3. James Stewart, "Calculus: Early Transcendentals", Cengage Learning, 8th Edition, New Delhi, 2015. [For Units II & IV - Sections 1.1, 2.2, 2.3, 2.5, 2.7 (Tangents problems only), 2.8, 3.1 to 3.6, 3.11, 4.1, 4.3, 5.1 (Area problems only), 5.2, 5.3, 5.4 (excluding net change theorem), 5.5, 7.1 - 7.4 and 7.8].

REFERENCES:

1. Anton. H, Bivens. I and Davis. S, "Calculus", Wiley, 10th Edition, 2016
2. Bali. N., Goyal. M. and Watkins. C., "Advanced Engineering Mathematics", Firewall Media (An imprint of Lakshmi Publications Pvt., Ltd.,), New Delhi, 7th Edition, 2009.
3. Jain. R.K. andlyengar. S.R.K., "Advanced Engineering Mathematics", Narosa Publications, New Delhi, 5th Edition, 2016.
4. Narayanan. S. and Manicavachagom Pillai. T. K., "Calculus" Volume I and II, S. Viswanathan PublishersPvt. Ltd., Chennai, 2009.
5. Ramana. B.V., "Higher Engineering Mathematics", McGraw Hill Education Pvt. Ltd, New Delhi, 2016.
6. Srimantha Pal and Bhunia. S.C, "Engineering Mathematics" Oxford University Press, 2015.
7. Thomas. G. B., Hass. J, and Weir. M.D, " Thomas Calculus ", 14th Edition, Pearson India, 2018.

CO's-PO's & PSO's MAPPING

CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3
CO1	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-
CO2	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-
CO3	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-
CO4	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-
CO5	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-
Avg	3	3	1	1	0	0	0	0	2	0	2	3	-	-	-

1 - low, 2 - medium, 3 - high, '-"- no correlation

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SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam - Tiruppur

(Accredited by NAAC, Under 2f and 12B status)

LESSON PLAN

Faculty Name : G.Jeevanantham
Department : CSE
Subject / Code : Matrices and Calculus
Academic Year : 2024-2025

Designation: Assistant Professor
Semester/ Year: I & I

S.No.	Proposed Date	Proposed Period	Details of Topic Covered	TA	Ref.	Actual Date	Actual Period	Remarks
			UNIT-I -MATRICES					
1	18.09.24	1,2	Eigenvalues and Eigenvectors of a real matrix - Characteristic equation	1	1	18.9.24	1,2	
2	19.09.24	7	Properties of Eigenvalues and Eigenvectors	1	1	19.09.24	7	
3	20.09.24	3,8	Properties of Eigenvalues and Eigenvectors	1	1	20.09.24	3,8	
4	20.09.24	6	Cayley - Hamilton theorem	1	1	21.09.24	6	
5	21.09.24	5	Diagonalization of matrices by orthogonal transformation	1	1	23.09.24	5	
6	23.09.24	4,6	Diagonalization of matrices by orthogonal transformation	1	1	24.09.24	4,6	
7	24.09.24	3,4	Reduction of a quadratic form to canonical form by orthogonal transformation	1	2	25.09.24	3,4	
8	25.09.24	3,4	Reduction of a quadratic form to canonical form by orthogonal transformation	1	2	25.09.24	3,4	
9	26.09.24	7	Nature of quadratic forms	1	2	26.09.24	7	
10	27.09.24	3,8	Nature of quadratic forms	1	2	27.09.24	3,8	
11	28.09.24	6	Applications: Stretching of an elastic membrane	1	2	28.09.24	6	
12	30.09.24	5	Applications: Stretching of an elastic membrane	1	2	30.09.24	5	
			UNIT II - DIFFERENTIAL CALCULUS					
13	01.10.24	4,6	Representation of functions	1	2	1.10.24	4,6	
14	03.10.24	7	Representation of functions	1	2	3.10.24	7	
15	04.10.24	3,8	Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules)	1	2	4.10.24	3,8	
16	05.10.24	6	Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules)	1	2	05.10.24	6	
17	07.10.24	5	Limit of a function - Continuity - Derivatives - Differentiation rules (sum, product, quotient, chain rules)	1	3	07.10.24	5	
18	07.10.24	5	Implicit differentiation	1	3	07.10.24	5	
19	08.10.24	1,2	Implicit differentiation	1	3	08.10.24	1,2	
20	09.10.24	7	Logarithmic differentiation	1	3	09.10.24	7	
21	14.10.24	3,8	Logarithmic differentiation	1	3	14.10.24	3,8	
22	15.10.24	6	Logarithmic differentiation	1	3	15.10.24	6	
23	15.10.24	5	Applications: Maxima and Minima of functions of one variable	1	3	15.10.24	5	
24	16.10.24	4,6	Applications: Maxima and Minima of functions of one variable	1	3	16.10.24	4,6	
			UNIT III - FUNCTIONS OF SEVERAL VARIABLES					
25	17.10.24	3,4	Partial differentiation	1	4	17.10.24	3,4	
26	18.10.24	3,4	Homogeneous functions and Euler's theorem	1	4	18.10.24	3,4	
27	19.10.24	6	Homogeneous functions and Euler's theorem	1	4	19.10.24	6	
28	21.10.24	3,6	Total derivative – Change of variables – Jacobians	1	4	21.10.24	3,6	
29	22.10.24	3,4	Total derivative – Change of variables – Jacobians	1	4	22.10.24	3,4	
30	22.10.24	3,4	Total derivative – Change of variables – Jacobians	1	4	22.10.24	3,4	
31	24.10.24	7	Partial differentiation of implicit functions	1	4	25.10.24	3,8	
32	25.10.24	3,8	Partial differentiation of implicit functions	1	5	26.10.24	6	
33	26.10.24	6	Taylor's series for functions of two variables	1	5	28.10.24	4,6	
34	28.10.24	4,6	Taylor's series for functions of two variables	1	5	29.10.24	3,4	
35	29.10.24	3,4	Applications: Maxima and minima of functions of two variables and Lagrange's method of undetermined multipliers	1	5	29.10.24	3,4	
36	04.11.24	4,6	Applications: Maxima and minima of functions of two variables and Lagrange's method of undetermined multipliers	1	5	04.10.24	4,6	
			UNIT IV - INTEGRAL CALCULUS					
37	05.11.24	3,4	Definite and Indefinite integrals - Substitution rule	1	2	5.11.24	3,4	
38	06.11.24	1,2	Definite and Indefinite integrals - Substitution rule	1	2	6.11.24	1,2	
39	07.11.24	7	Techniques of Integration: Integration by parts, Trigonometric integrals	1	2	7.11.24	7	
40	08.11.24	3,8	Techniques of Integration: Integration by parts, Trigonometric integrals	1	2	08.11.24	3,8	

41	09.11.24	6	Techniques of Integration Integration by parts Trigonometric integrals	1	2	09.11.24	6	
42	11.11.24	2,4	Trigonometric substitutions. Integration of rational functions by partial fraction	1	2	11.11.24	2,4	
43	12.11.24	1,2	Trigonometric substitutions. Integration of rational functions by partial fraction	1	5	12.11.24	1,2	
44	13.11.24	7	Trigonometric substitutions. Integration of rational functions by partial fraction	1	5	13.11.24	7	
45	14.11.24	3,8	Integration of irrational functions	1	5	14.11.24	3,8	
46	15.11.24	4,6	Improper integrals	1	5	15.11.24	4,6	
47	16.11.24	7	Applications Hydrostatic force and pressure, moments and centres of mass	1	5	16.11.24	7	
48	18.11.24	4,6	Applications Hydrostatic force and pressure, moments and centres of mass	1	5	18.11.24	4,6	
UNIT V - MULTIPLE INTEGRALS								
49	19.11.24	3,4	Double integrals – Change of order of integration	1	1	19.11.24	3,4	
50	20.11.24	1,2	Double integrals in polar coordinates	1	1	20.11.24	1,2	
51	22.11.24	7	Double integrals in polar coordinates	1	1	22.11.24	7	
52	23.11.24	3,8	Double integrals in polar coordinates	1	1	23.11.24	3,8	
53	25.11.24	4,6	Area enclosed by plane curves	1	1	25.11.24	4,6	
54	26.11.24	3,4	Area enclosed by plane curves	1	2	26.11.24	3,4	
55	27.11.24	1,2	Triple integrals – Volume of solids	1	2	27.11.24	1,2	
56	28.11.24	7	Triple integrals – Volume of solids	1	2	28.11.24	7	
57	29.11.24	3,8	Change of variables in double and triple integrals	1	2	29.11.24	3,8	
58	30.11.24	6	Change of variables in double and triple integrals	1	2	30.11.24	6	
59	02.12.24	3,8	Applications Moments and centres of mass, moment of inertia	1	2	02.12.2024	3,8	
60	03.12.24	4,6	Applications Moments and centres of mass, moment of inertia	1	2	03.12.24	4,6	

Text books :

1. Kreyszig E. "Advanced Engineering Mathematics". John Wiley and Sons.10th Edition, New Delhi, 2016
2. Grewal B S., "Higher Engineering Mathematics". Khanna Publishers. New Delhi.44th Edition . 2018
3. James Stewart, " Calculus Early Transcendentals", Cengage Learning, 8th Edition, New Delhi, 2015. [For Units II & IV - Sections 1.1. 2.2, 2.3, 2.5, 2.7 (Tangents problems only), 2.8, 3.1 to 3.6, 3.11, 4.1, 4.3, 5.1 (Area problems only), 5.2, 5.3, 5.4 (excluding net change theorem), 5.5, 7.1 - 7.4 and 7.8]

Reference books (Ref):

1. Anton. H, Bivens. I and Davis. S. "Calculus", Wiley, 10th Edition, 2016
2. Narayanan. S. and Manicavachagom Pillai. T. K., "Calculus" Volume I and II, S.Viswanathan Publishers Pvt. Ltd., Chennai, 2009.
3. Ramana. B.V., "Higher Engineering Mathematics", McGraw Hill Education Pvt. Ltd, New Delhi, 2016.
4. Thomas. G. B., Hass. J. and Weir. M.D, "Thomas Calculus", 14th Edition, Pearson India,2018.

Teaching Aids (TA):

1. Black Board with Chalk
2. Overhead Projector
3. LCD Projector

	Prepared by	Verified by	Authorized by
Sign			
	Faculty	HOD	Principal

SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Approved by AICTE, New Delhi
Affiliated to Anna University, Chennai

TEST PLAN FOR SUBJECT

Subject : Matrices and Calculus Faculty: G. Jeevanantham.

Semester : I Year: I

Department : CSE

S. No.	Description	Planned Date/Month	Actual Conducted Date / Month	Remarks
1	Internal Examination - I	12.11.24	12.11.24	—
2.	Internal Examination - II	27.11.24	27.11.24	—
3.	Model Examination - I	11.12.24	11.12.24	—

	Prepared By	Approved By
Sign:	[signature]	[signature]
Name:	G. Jeevanantham.	M. Sathya
	Faculty	HoD

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RESULT ANALYSIS OF TEST

Subject : Matrices and Calculus   Date : 13.11.24

Class : I Sec-B.   Department : CSE

Semester : I

Exam details & date : Internal Examination I, 12.11.24.

Faculty : G. Jeevanantham.

Number of students : 55

No. of students attended : 54

No. of students absent : 01

No. of students passed : 08

No. of students failed : 46

Percentage of failures : 83%

RESULT DATA:

Marks	0-25	26-50	51-75	76-90	91-100
No. of Students	28	18	10	4	4

	Prepared By	Approved By
Sign:	[signature]	[signature]
Name:	G. Jeevanantham.	M. Sathya
	Faculty	HoD

SASURIE INSTITUTIONS

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Affiliated to Anna University, Chennai

CORRECTIVE ACTION REPORT

Department : CSE

Year : I

Semester : I

Subject : Matrices and Calculus.

NON CONFORMANCE REPORT
Expected Result 100% and received 17% Result Date: 13.11.24 Faculty Sign
ROOT CAUSE ANALYSIS
They did mistake in formulas and steps in the given Problems Date: 13.11.24 Faculty Sign
CORRECTIVE ACTION
To give an imposition for the students. Date: 13.11.24. Faculty Sign
VERIFICATION OF CORRECTIVE ACTION
verified Date: 13/11/2024 Faculty Sign

	Prepared By	Approved By
Sign:		M. Sathya
Name:	G. Jeevan Ram	M. Sathya
	Faculty	HoD

SASURIE INSTITUTIONS

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RESULT ANALYSIS OF TEST

Subject : Matrices and Calculus   Date : 28.11.2024.

Class : I SecB.   Department : CSE

Semester : I

Exam details & date : Internal Examination II, 27.11.2024.

Faculty : G. Jeevanantham.

Number of students : 55

No. of students attended : 48

No. of students absent : 07

No. of students passed : 11

No. of students failed : 37

Percentage of failures : 77%

RESULT DATA:

Marks	0-25	26-50	51-75	76-90	91-100
No. of Students	16	21	10	1	—

	Prepared By	Approved By
Sign:	[signature]	[signature]
Name:	G. Jeevanantham.	M. Sathya
	Faculty	HoD

SASURIE INSTITUTIONS

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CORRECTIVE ACTION REPORT

Department : CSE

Year : I

Semester : I

Subject : Matrices and Calculus.

NON CONFORMANCE REPORT Expected Result 100%. and received. 23% result. Date: 28.11.2024 Faculty Sign
ROOT CAUSE ANALYSIS They did mistake in formulas and steps. Date: 28.11.2024 Faculty Sign
CORRECTIVE ACTION To give an Assignment. Date: 28.11.2024 Faculty Sign
VERIFICATION OF CORRECTIVE ACTION Verified Date: 28/11/2024 Faculty Sign

	Prepared By	Approved By
Sign:		H. [illegible]
Name:	G. Jeevanantham	M. Sathya
	Faculty	HoD

SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING
Approved by AICTE, New Delhi
Affiliated to Anna University, Chennai

RESULT ANALYSIS OF TEST

Subject : Matrices and Calculus Date : 14.12.2024.

Class : I sec B. Department : CSE.

Semester : I

Exam details & date : Model Examination I, 11.12.2024

Faculty : G. Jeevanantham.

Number of students : 55

No. of students attended : 46

No. of students absent : 09

No. of students passed : 15

No. of students failed : 31

Percentage of failures : 67

RESULT DATA:

Marks	0-25	26-50	51-75	76-90	91-100
No. of Students	10	23	09	03	01

	Prepared By	Approved By
Sign:	[signature]	[signature] 14/12/2024
Name:	G. Jeevanantham.	
	Faculty	HoD

SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Approved by AICTE, New Delhi
Affiliated to Anna University, Chennai

CORRECTIVE ACTION REPORT

Department : CSE

Year : I

Semester : I

Subject : Matrices and Calculus.

NON CONFORMANCE REPORT Expected Result is 100%. Received 33% Result. Date: 14.12.2024 Faculty Sign
ROOT CAUSE ANALYSIS They did Mistake in Methods and Formulae Date: 14.12.2024 Faculty Sign
CORRECTIVE ACTION To given an Imposition. Date: 14.12.2024. Faculty Sign
VERIFICATION OF CORRECTIVE ACTION Verified Date: Faculty Sign

	Prepared By	Approved By
Sign:		
Name:	G. Jeevanand Ram	M. Sathya (G. Sathya)
	Faculty	HoD

SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Approved by AICTE, New Delhi
Affiliated to Anna University, Chennai

QUALITY OBJECTIVE MONITORING RECORD

Department : CSE

Year : I

Semester : I

Subject : Matrices and Calculus.

S.No	Quality Objective	Internal Test-I		Internal Test-II		Model Test-I	
		Expecting result	Obtained result	Expecting result	Obtained result	Expecting Result	Obtained result
1.	To develop the use of Matrix algebra techniques that is needed by Engineers for practical Applications.	100%	17%	100%	23%	100%	

	Prepared By	Approved By
Sign:		
Name:	G. Jeevanan Kum.	
	Faculty	HoD

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SASURIE

SASURIE INSTITUTIONS

COLLEGE OF ENGINEERING

Vijayamangalam, Tiruppur

WEBPORTAL ENTRY I CONSOLIDATE

ACADEMIC YEAR :2024-2025
YEAR & SEM : I/I
DEPARTMENT :I CSE
SUBJECT & CODE : MATRICES AND CALCULUS (MA3151)
NAME OF THE FACULTY : G.JEEVANANTHAM

DATE : 08.11.2024

S.No	Register Number	Name	Total number of hours	Attended hours	MATRICES AND CALCULUS									
					Assignment 1	Slip Test-I	Slip Test-II	Slip Test-III	B -I(20)	B-II (20)	B MARK TOTAL (40)	MARKS(60)	ASS(40)	TOTAL (100)
1	732424104062	Mathavan.V	45	35	19	13	19	AB	13	19	32	48	38	86
2	732424104063	Merlin jenisha Mery.M	45	44	19	12	18	14	14	18	32	48	38	86
3	732424104064	Methini.L	45	41	19	9	8	AB	9	8	17	26	38	64
4	732424104065	Mohammad Fayaz.S	45	41	18	11	4	AB	11	4	15	23	36	59
5	732424104066	Mohammed Suhail.S	45	44	19	15	9	8	15	9	24	36	38	74
6	732424104067	Monisha.N.P	45	40	19	13	13	19	13	19	32	48	38	86
7	732424104069	Nathiya.R	45	40	19	14	17	AB	14	17	31	47	38	85
9	732424104070	Naveenkumar.R	45	30	17	AB	AB	AB	0	0	0	0	34	34
8	732424104071	Nivetha.P	45	41	19	17	11	19	17	19	36	54	38	92
10	732424104072	Pradeepa. L	45	40	19	16	19	AB	16	19	35	53	38	91
11	732424104073	Prapeena.B	45	38	19	14	7	19	14	19	33	50	38	88
12	732424104074	Prema.K	45	41	19	18	14	18	18	18	36	54	38	92
13	732424104075	Priyadharsan.V	45	40	19	14	16	AB	14	16	30	45	38	83
14	732424104076	Ragul.O	45	35	19	11	6	16	11	16	27	41	38	79
15	732424104077	Ragul.R	45	38	18	9	5	6	9	6	15	23	36	59
16	732424104078	Rajapriyan.K	45	37	19	11	17	14	17	14	31	47	38	85

17	732424104079	Sabarinathan. K	45	41	19	17	19	15	19	17	36	54	38	92
18	732424104080	Sabarish.V	45	40	19	2	5	9	5	9	14	21	38	59
20	732424104081	Sanjay.M	45	40	19	9	7	AB	9	7	16	24	38	62
19	732424104082	Sanjay Kumar.R	45	40	19	7	4	11	7	11	18	27	38	65
21	732424104083	Sanmathi.S	45	44	19	14	17	13	14	17	31	47	38	85
22	732424104084	Sanofar. M	45	41	19	10	7	19	10	19	29	44	38	82
23	732424104085	Santhiya.S.V	45	37	19	16	6	13	16	13	29	44	38	82
24	732424104086	Sarathi.G	45	40	17	16	4	8	16	8	24	36	34	70
25	732424104087	Saravanan. M	45	37	19	11	9	9	11	9	20	30	38	68
26	732424104088	Sarmitha.P.V.K	45	41	19	16	19	20	19	20	39	59	38	97
27	732424104089	Sasvitha. S	45	38	19	11	18	18	18	18	36	54	38	92
28	732424104090	Senthamizh.J	45	38	19	10	7	16	10	16	26	39	38	77
29	732424104091	Shalika.S	45	41	19	15	14	AB	15	14	29	44	38	82
30	732424104092	shamprasanth.S	45	41	19	7	5	AB	7	5	12	18	38	56
31	732424104093	Shyam Siddharth.S	45	24	19	AB	AB	AB	0	0	0	0	38	38
32	732424104094	Sidharsan.M	45	38	19	11	12	13	12	13	25	38	38	76
33	732424104095	Soundarya.S.T	45	42	17	14	16	AB	14	16	30	45	34	79
34	732424104096	Sowmiya.B	45	42	19	9	5	14	9	14	23	35	38	73
35	732424104097	Sowmiya.M	45	45	19	18	18	14	18	8	26	39	38	77
36	732424104098	Sriharishma.R	45	35	19	15	19	12	15	19	34	51	38	89
37	732424104099	Sriram.K	45	29	17	6	3	AB	6	3	9	14	34	48
38	732424104100	Srishanth.K	45	44	19	12	7	11	12	11	23	35	38	73
39	732424104101	Stephy Rose Manna.A	45	40	19	16	19	19	19	19	38	57	38	95
40	732424104102	Sujith.S	45	41	19	8	7	6	8	7	15	23	38	61
41	732424104103	Suthishna.S	45	34	19	12	8	11	12	11	23	35	38	73

42	732424104104	Swetha.V	45	44	19	16	19	20	19	20	39	59	38	97
43	732424104105	Tharunkumar.K	45	40	19	6	2	10	6	10	16	24	38	62
44	732424104106	Thenmozhi. M	45	40	17	6	5	AB	6	5	11	17	34	51
45	732424104107	Thirumurugan.M	45	37	17	9	8	10	9	10	19	29	34	63
46	732424104109	Varshini. N	45	41	19	15	17	15	15	17	32	48	38	86
47	732424104111	Velusamy.M	45	41	19	14	7	17	14	17	31	47	38	85
48	732424104112	Venkatraj.R	45	31	18	3	0	5	3	5	8	12	36	48
49	732424104113	Vidhyavarshini.M	45	35	17	AB	4	AB	4	0	4	6	34	40
50	732424104114	Vignesh.M	45	39	18	5	5	6	5	6	11	17	36	53
51	732424104115	Vimalesh. R. K	45	44	19	16	18	17	17	18	35	53	38	91
52	732424104116	Vinthia varshini.S	45	44	19	17	17	20	17	20	37	56	38	94
53	732424104117	Vishal.M	45	41	15	14	AB	13	14	13	27	41	30	71
54	732424104118	Yogalakshmi.G	45	44	19	AB	8	13	8	13	21	32	38	70
55	732424104118	Yuvashri.S	45	41	19	14	13	AB	14	13	27	41	38	79

	Prepared by	Verified by	Approved by
Sign	08/11/2024	M. 8/11/2024	8-11-2024
Name	G.JEEVANANTHAM	M.SATHYA	Dr.M.Vijayakumar
	Faculty	HoD	Principal

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SASURIE INSTITUTIONS

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, Tiruppur.

Set : B

Internal Examination - I		Date/Session	12.11.24 / FN	Marks	50
Course code	MA3151	Course Title	MATRICES AND CALCULUS		
Regulation	2021	Duration	1.30 Hours	Academic Year	2024-2025
Year	I	Semester	I	Department	Common to all Branch

COURSE OUTCOMES	
CO1:	Use the matrix algebra methods for solving practical problems.
CO2:	Apply differential calculus tools in solving various application problems
CO3:	Able to use differential calculus ideas on several variable functions.
CO4:	Apply different methods of integration in solving practical problems.
CO5:	Apply multiple integral ideas in solving areas, volumes and other practical problems.

Q.No.	Questions	CO	BTS
	PART A (Answer all the Questions 10 x 2 = 20 Marks)		
1	If 2,-1,-3 are the Eigen value of the matrix A,then find the Eigen value of A^2-2I.	CO1	U
2	If 2 and 3 are the two eigenvalues of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$ then find the value of b.	CO1	A
3	Write down the matrix corresponding to the quadratic form $2x_1^2+5x_2^2+4x_1x_2+2x_3x_1$.	CO1	R
4	If the sum of two Eigen values and trace of a 3x3 matrix A are equal, find the value of $\|A\|$.	CO1	U
5	Find the Eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.	CO1	U
6	Find the derivative $y=(x^3-1)^{100}$	CO2	U
7	check whether $\lim_{x\to 1} \frac{3x+9}{\|x+3\|}$ exists	CO2	R
8	Find the critical points of $y=5x^3-6x$	CO2	A

9	State Mean value theorem	CO2	R
10	Evaluate the limit for $\lim_{x\to -2}\frac{x+2}{x^5+8}$	CO2	A
	PART B (Answer all the Questions 2 x 15 =30 Marks)		
11a	i) Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ ii) Verify the Cayley-Hamilton theorem and also find A^4 for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$	CO1	U
	OR		
11b	Reduce the quadratic form into the canonical by using orthogonal transform $6x^2{}_1 + 3x^2{}_2 + 3x^2{}_3 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ and also find Rank, signature, Index	CO1	E
12a	i) Find an equation of the tangent line to the hyperbolay $= 4x - 3x^2$ at (3,1) ii) If $f(x) = \sqrt{x}$ find $f^{ll}(x)$	CO2	A
	OR		
12b	i) Determine whether $f'(0)$ exist or not for the given function $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$ ii) Find the domain at which the function continuous and differentiable. $f(x) = \|x\|$	CO2	R

Course Faculty
P.Sivaranjani
(Mrs.P.Sivaranjani)

HoD
M.Sathya 5/11/2024
(Mrs.M.Sathya)

Principal
(Dr.M.Vijayakumar)

SASURIE COLLEGE OF ENGINEERING

INTERNAL TEST - I

ANSWER KEY

DEPARTMENT : B.E - ECE & EEE

DATE: 12-11-2024

SUBJECT : MATRICES AND CALCULUS

SUBJECT CODE: MA3151

PART - A

$A = 2, -1, -3$.

$A^2 = 4, 1, 9$

$A^2 - 2I$ is $2, -1, 7$

i. $\lambda_3 = 1$

ii. $b = 1$

i. $\begin{bmatrix} 2 & 2 & 1 \\ 2 & 5 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

i. Trace of A = sum of two eigen value

$= \lambda_1 + \lambda_2$

$\lambda_3 = 0$

$|A| = 0$

i. The characteristic equation $\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$

ii. $s_1 = 7$

iii $s_2 = 0$

iv. $s_3 = -36$

v. $\lambda = 3, 6, -2$

2. i. $y' = 100(x^3-1)^{99}(3x)^2$

ii. $= 300x^2(x^2-1)^{99}$

7. i. $\lim_{t \to 2} \frac{3t+9}{|t+3|}$

ii. Does not exist.

3. i. $y' = 15x^2 - 6$

ii. critical point $x = \pm\sqrt{2/5}$

7. i. The $f'(x)$ is continuous of $[a,b]$ and the $f'(x)$ is differentiable on (a,b) is called mean value Theorem.

ii. $f(c) = \frac{f(a)-f(b)}{f(ab)}$

0. i. $0/0$

ii. It is an indefined form

iii. Using LH rule

iv. $1/12$

PART-B

1. a

i. The characteristic equation are $|A-\lambda I| = 0$.

$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$.

ii. $s_1 = 18$

iii $s_2 = 99$

iv. $s_3 = 162$

v. The eigen values are $\lambda = 3, 9, 6$

vi. The eigen vector for $\lambda = 9$ is $\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

vi. The eigen vector for $\lambda = 6$ is $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

vii. The eigen vector for $\lambda = 3$ is $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

i. The characteristic equation are $|A - \lambda I| = 0$.

i.e. $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

ii. $S_1 = 1+2+3 = 6$

iii. $S_2 = 5+2-2 = 5$

iv. $S_3 = 5-10 = -5$

v. $A^3 - 6A^2 + 5A + 5I = 0$.

vi. $A^2 = \begin{bmatrix} 6 & 7 & 6 \\ 7 & 9 & 7 \\ 6 & 7 & 11 \end{bmatrix}$

vii. $A^2 A = \begin{bmatrix} 26 & 32 & 31 \\ 32 & 39 & 37 \\ 31 & 37 & 46 \end{bmatrix}$

viii. Hence cayley Hamilton theorem is verified.

ix. $A^4 = \begin{bmatrix} 121 & 147 & 151 \\ 147 & 179 & 182 \\ 151 & 182 & 206 \end{bmatrix}$

i. $y' = 4 - 6x$

ii $y' = -14$.

iii. To find the tangent equation is $(y - y_1) = m(x - x_1)$.

iv. $y = -14x + 43$.

v. The tangent equation of the point $(3, 1)$.

a.
ii. i. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

ii. $f'(x) = \lim_{h \to 0} \left(\dfrac{\sqrt{x+h} - \sqrt{x}}{h} \times \dfrac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right)$

iii. The $f'(x) = -1/2\sqrt{x}$

1.b.
i. i. $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

ii. $S_1 = 12$

iii $S_2 = 36$

iv. $S_3 = 32$

v. The eigen values are $\lambda = 8, 2, 2$.

vi. The eigen vector of $\lambda = 8$ is $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

vii. The eigen vector of $\lambda = 2$ is $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

viii. The eigen vector of $\lambda = 2$ is $\begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$

ix. $D = N^T A N = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

x. $C = Y^T D Y = 8y_1^2 + 2y_2^2 + 2y_3^3$

xi. Index (P) = 3

xii. Rank (r) = 3

xiii. Signature $= 2p - r$

$= 2(3) - 3$

$= 6 - 3$

$= 3$.

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Register Number:											

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SASURIE COLLEGE OF ENGINEERING

SASURIE INSTITUTIONS

Vijayamangalam, Tiruppur.

Internal Examination - II			Date/Session	27.11.2024/ FN	Marks	50
Course code	MA3151	Course Title	MATRICES AND CALCULUS			
Regulation	2021	Duration	1.30 Hours	Academic Year	2024-2025	
Year	I	Semester	I	Department	Common to All Branches.	

COURSE OUTCOMES	
CO1:	Use the matrix algebra methods for solving practical problems.
CO2:	Apply differential calculus tools in solving various application problems.
CO3:	Able to use differential calculus ideas on several variable functions.
CO4:	Apply different methods of integration in solving practical problems.
CO5:	Apply multiple integral ideas in solving areas, volumes and other practical problems.

Q.No.	Questions	CO	BTS
	PART - A (Answer all the Questions 10 x 2 = 20 Marks)		
1	If $y = x\log\left(\frac{x-1}{x+1}\right)$, then find $\frac{dy}{dx}$.	CO2	R
2	Prove that $\lim_{x\to 0}\frac{\lvert x\rvert}{x}$ does not exist.	CO2	R
3	Find the slope of the circle $x^2+y^2=25$ at (3,-4).	CO2	E
4	Evaluate the limit $\lim_{x\to 1}\frac{x^2-4x}{x^2-3x-4}$.	CO2	R
5	Find $\frac{\partial^2 w}{\partial x\partial y}$, if $w = xy + \frac{e^x}{y^2+1}$.	CO3	R
6	Write Euler's theorem on homogeneous function.	CO3	U
7	If $u = x^3+y^3$ when $x = \cos t$, $y = b \sin t$ then find $\frac{du}{dt}$.	CO3	R
8	Find the stationary point $f(x,y) = x^2 - xy + y^2 - 2x + y$	CO3	R
9	If $u = \frac{2x-y}{2}$ and $v = \frac{y}{z}$ then find $\frac{\partial(u,v)}{\partial(x,y)}$.	CO3	R

10	Find $\frac{dy}{dx} = x^3 + y^3 = 3axy$.	CO3	R
	PART - B (Answer all the Questions 2 x 15 = 30 Marks)		
11.a	(i) Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2+1$ on the interval [-2,3]	CO2	A
	(ii) Find the local maximum and local minimum values of $f(x) = \sqrt{x} + \sqrt[4]{x}$ using first and second derivatives test,	CO2	R
	OR		
11.b	(i) Using Taylor's series, expand $f(x,y) = x^2y + \sin y + e^x$ upto the second degree terms at the point $(1,\pi)$.	CO2	A
	(ii) If $u = \log(x^2 + y^2 + z^2)$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2+y^2+z^2}$.	CO2	U
12.a	(i) If $u = \sin^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.	CO3	U
	(ii) Find the maximum and minimum values of $f(x,y) = x^2 - xy + y^2 - 2x + y$.	CO3	R
	OR		
12.b	Find the dimensions of the rectangular box without a top of maximum Capacity, whose surface area is 108 sq.cm.	CO3	A

25/11/24
Course Faculty
(Mr.G.Jeevanantham)

M.Sathya 25/11/2024.
HoD
(Mrs.M.Sathya)

Principal
(Dr.M.Vijayakumar)

SASURIE COLLEGE OF ENGINEERING

INTERNAL TEST - II

ANSWER KEY

DEPARTMENT: B.E- ECE & EEE DATE: 27-11-2024

SUBJECT: MATRICES AND CALCULUS SUBJECT CODE: MA3151

PART - A.

i. $\frac{dy}{dx} = x \log\left(\frac{(x-1)}{(x+1)}\right)$

ii. $UV = UV' + VU'$

iii. $\frac{dy}{dx} = \frac{x^2+x}{(x-1)} + \log\left(\frac{x-1}{x+1}\right)$

i. $\lim_{x \to 0^-} \frac{-(-1)}{-1} = \frac{1}{-1} = -1$

ii $\lim_{x \to 0^+} \frac{x}{x} = \frac{1}{1} = 1.$

iii The left side limit is not equal to right side limit.

iv. $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

i. $y - y_1 = m(x - x_1)$

i. $3x - 4y + 7 = 0$

ii. $\frac{-x}{(25-x^2)^2}$

i. $\frac{(1)^2 - 4(1)}{(1)^2 - 3(1) - 4} = \frac{1-4}{1-3-4} = \frac{-3}{-6} = 1/2$

ii $= \frac{1}{2}$

i. $x \frac{du}{dx} + y \frac{du}{dx} = nu.$

ii. z is a homogenous function of degree n at (x, y).

1\. i. $\frac{du}{dt} = \frac{du}{dx} \frac{dx}{dt} + \frac{du}{dy} \frac{dy}{dt}.$

ii. $\frac{du}{dt} = 3 \sin t \cos t \ (b^3 \cos t - a^3 \sin t).$

8\. i. $\frac{df}{dx} = 2x - y - 2 = 0 \rightarrow (1)$

ii. $\frac{df}{dy} = -x + 2y + 1 = 0 \rightarrow (2)$

iii. The stationary points are (1, 0).

9\. i. $\frac{d(u,v)}{d(x,y)} = \begin{vmatrix} \frac{du}{dx} & \frac{du}{dy} \\ \frac{dv}{dx} & \frac{dv}{dy} \end{vmatrix}$

ii. $\begin{vmatrix} 1 & -1/2 \\ 0 & -1/2 \end{vmatrix}$

10\. i. $\frac{dz}{dx} = 3x^2 + 0 - 3ay = 3x^2 - 3ay.$

ii. $\frac{dz}{dy} = 3y^2 - 3ax.$

iii. $\frac{d^2z}{dxdy} = \frac{d}{dx}(3y^2 - 3ax) = -3a.$

PART - B.

1. i. $f'(x) = 12x(x^2 - x - 2)$

ii. $f'(x) = 0$

iii. $x = 0 \quad x = -1 \quad x = 2.$

iv. $F(0) = 3(0)^4 - 4(0)^3 - 12(0)^2 + 1$

$= +1$

v. $F(-1) = 3(-1)^4 - 4(-1)^3 - 12(-1)^2 + 1$

$= 3 + 4 - 12 + 1$

$= -4$

vi. $F(2) = 3(2)^4 - 4(2)^3 - 12(2)^2 + 1$

$= 3(16) - 4(8) - 12(4) + 1$

$= -31.$

vii. The interval are $[-2, 3]$

viii. $f(-2) = 33$

ix. $f(3) = 28.$

i. $f(x) = (x)^{1/2} + (x)^{1/4}$

ii. $f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} + \frac{1}{4} x^{\frac{1}{4}-1}$

iii. $f'(x) = \frac{1}{2} x^{-3/4} (2x^{1/4} + 1)$

iv. critical point $f'(x) = 0.$

v. $x = (1/16)$

i. $f''(x) = 1/2(-1/2) x^{\frac{-1}{2}-1} + 1/4(-3/4) x^{-3/4-1}$

ii. $-1/4 \, x^{-7/4} (4x^{1/4} + 3)$

iii. $= -1/16 (128)(5)$

x. $= \frac{-\cancel{128}\ \cancel{64}\ 8}{\cancel{16}\ 8}$

x. $= -8(5)$

i. The local maximum $f''(x) = -40 < 0$

ii. $3/4 > 0$ It is the local maximum.

b.

i) i. $f(x,y) = x^2y + \sin y + e^x$

ii. $f(a,b) = (1,\pi)$

iii. $f(x-a) = (x-1)$

iv. $f(x-b) = (x-\pi)$

v. $f(x,y) = x^2y + \sin y + e^x$; $f(1,\pi) = \pi + e$.

vi. $f_x = 2xy + e^x$; $f_x(1,\pi) = 2\pi + e$.

vii. $f_y = x^2 + \cos y$; $f_y(1,\pi) = 0$

viii. $f_{xx} = 2y + e^x$; $f_{xx}(1,\pi) = 2\pi + e$.

xi. $f_{yy} = -\sin y$; $f_{yy}(1,\pi) = 0$

x. $f_{xy} = 2x$; $f_{xy}(1,\pi) = 2$.

xi. $f(x,y) = (\pi+e) + (x-1)(2\pi+e) + \frac{1}{2}(x-1)^2(2\pi+e) + 2(x-1)(y-\pi) + \cdots$

a

i. i. $\sin u = z$

ii. $\dfrac{x^2(1+y^3/x^3)}{(1+y/x)}$

iii. $x\dfrac{dz}{dx} + y\dfrac{dz}{dy} = nz$

iv. $x\dfrac{du}{dx} + y\dfrac{du}{dy} = 2\tan u$.

ii. i. Partial differentiate with respect to x and y

ii $\dfrac{df}{dx} = 2x - y - 2$

iii $\dfrac{df}{dy} = -x + 2y + 1$

iv. $x = 1$ $y = 0$.

v. A is +ve , $AC - B^2$ is +ve $f(x,y)$ is minimum at $(1,0)$.

b.

i. i. $xy + 2yz + 2zx - 108$.

ii. $F(x,y,z) = f(x,y,z) + \lambda\, g(x,y,z)$.

iii. $\dfrac{dF}{dx} = yz + \lambda(y+2z)$

iv. $\dfrac{dF}{dy} = xz + \lambda(x+2z)$

v. $\dfrac{dF}{dz} = xy + \lambda(2y+2x)$.

vi. $\frac{dF}{dx} = 0$

vii $\frac{dF}{dy} = 0$

viii $\frac{dF}{dz} = 0$

xi. $y = x$

x. $z = y/2$.

xi. $3y^2 = 108$

xii. $y^2 = \frac{108}{3}$

xiii. $y^2 = 36$

xiv. $y = 6$

xvi. $x = 6$

xv. $z = y/2 \Rightarrow 6/2 \Rightarrow 3$.

i. $xy + 2yz + 2zx = 0$.

ii. $F(x,y,z) = f(x,y,z) + \lambda g(x,y,z)$

iii. $\frac{dF}{dx} = 2y + 2z - 0$

iv. $\frac{dF}{dy} = 2z + 2y + \lambda yz$

v. $\frac{dF}{dz} = 2y + 2z + \lambda(xy)$.

ii. $y = x$.

iii. $z = x/2$.

viii. $x = 4$

xi. $y = 4$.

x. $z = x/2$.

xi. $z = 4/2$

xii. $z = 2$.

Register Number:												

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, Tiruppur

Model Examination-I		Date /Session	11.12.2024 /FN	Marks	100
Course code	MA3151	Course Title	MATRICES AND CALCULUS		
Regulation	2021	Duration	3 Hours	Academic Year	Regulation
Year	I	Semester	I	Department	Year

COURSE OUTCOMES

CO1:	Explain the fundamental concepts of advanced algebra and their role in modern mathematics and applied contexts
CO2:	Demonstrate accurate and efficient use of advanced algebraic techniques.
CO3:	Apply the concept of random processes in engineering disciplines.
CO4:	Understand the fundamental concepts of probability with a thorough knowledge of standard distributions that can describe certain real-life phenomenon.
CO5:	Understand the basic concepts of one and two dimensional random variables and apply them to model engineering problems.

Q.No.	Questions	CO	BTS
	PART A (Answer all the Questions 10 x 2 = 20 Marks)		
1	Find the eigen values of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.	CO1	A
2	Write the quadratic form corresponding to the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -2 & -4 \\ 3 & -4 & -3 \end{pmatrix}$.	CO1	U
3	The eigenvalues and the corresponding eigenvectors of a 2 × 2 matrix is given by $\lambda_1 = 8$; $x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\lambda_2 = 4$; $x_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find the corresponding matrix.	CO1	A
4	Prove that $x^2 - y^2 + 4z^2 + 4xy + 2yz + 6xz$ is indefinite.	CO1	U
5	Sketch the graph of the function f(x) = $\begin{cases} x^2 & if -2 \le x \le 0 \\ 2 - x & if\ 0 < x \le 2 \end{cases}$	CO2	U
6	Find the domain of the function $f(x) = \frac{2x^3-5}{x^2+x-6}$.	CO2	U
7	Differentiate $y = x\tan(\sqrt{x})$ with respect to x.	CO2	A
8	If $u = x - y, v = y - z, w = z - x$, then find the jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	CO3	A
9	If $z = x^2 + y^2$, and $x = t^2$, $y = 2at$, find $\frac{dz}{dt}$.	CO3	U
10	Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.	CO3	A
	PART B (Answer all the Questions 5*16 = 80 Marks)		
11(a)	Reduce the Quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$ to a canonical form by orthogonal reduction and find the nature, rank, index & signature. (16)	CO1	A
	OR		
11(b)	(i) Find the eigen values & eigen vectors of the matrix	CO1	A

	$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$.(8)		
	(ii) Using Cayley Hamilton theorem, find the inverse of the matrix $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$.(8)	CO1	A
12(a)	(i) Using Cayley Hamilton theorem, find the A^4 if $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{pmatrix}$. (8)	CO1	U
	(ii) Find the eigen values & eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.(8)	CO1	A
	OR		
12(b)	Obtain an orthogonal transformation which will transform the quadratic form $Q = 2x_1x_2 + 2x_2x_3 + 2x_1x_3$ to canonical form. (16)	CO1	A
13(a)	(i) Discuss the curve $f(x) = x^4 - 4x^3$ for the points of inflection, and local maxima & local minima. (8)	CO2	U
	(ii) For the what values of a and b is $f(x) = \begin{cases} -2 & x \le -1 \\ ax - b & -1 < x < 1 \\ 3 & x \ge 1 \end{cases}$ continuous at every x?. (8)	CO2	U
	OR		
13(b)	(i) Find the local maxima and minima of $f(x) = \sqrt{x} - \sqrt[4]{x}$. (8)	CO2	U
	(ii) Use logarithmic differentiation to differentiate $y = \frac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$.(8)	CO2	A
14(a)	(i) Find the equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $(1, e/2)$. (8)	CO2	A
	(ii) Differential of $(2x+3)^5\,(x^3 - x + 1)$. (8)	CO2	A
	OR		
14(b)	(i) If $u = \log(x^2 + y^2 + z^2)$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. (8)	CO3	U
	(ii) Find the maxima & minima for the given function $f(x,y) = x^3y^2(1 - x - y)$. (8)	CO3	U
15(a)	(i) Expand $e^x \cos y$ in a series of powers of x and y as far as the terms of the thirds degree. (8)	CO3	A
	(ii) A rectangular open box at the top is constructed so as have a volume of 108 cubic meters Find the dimension of the boxes that require the least material for its construction.(8)	CO3	U
	OR		
15(b)	(i) If $u = \sin^{-1}\left(\frac{x^3 - y^3}{x+y}\right)$ the prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. (8)	CO3	U
	(ii) Using Taylor's series expansion of the function $F(x,y) = x^2y + \sin y + e^x$ upto the second degree terms at the point $(1,\pi)$. (8)	CO3	A

Course Faculty 10/12/2024 — (Mr.G.Jeevanantham)

HoD M.Sathya 10/12/2024 — (Mrs.M.Sathya)

Principal — (Dr.M.Vijayakumar)

SASURIE COLLEGE OF ENGINEERING

MODEL EXAM - I

ANSWER KEY.

DATE : 11/12/2024

DEPARTMENT : ECE & EEE

SUBJECT CODE : MA3151

PART-A

2 Marks :

i) $A^{-1} = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

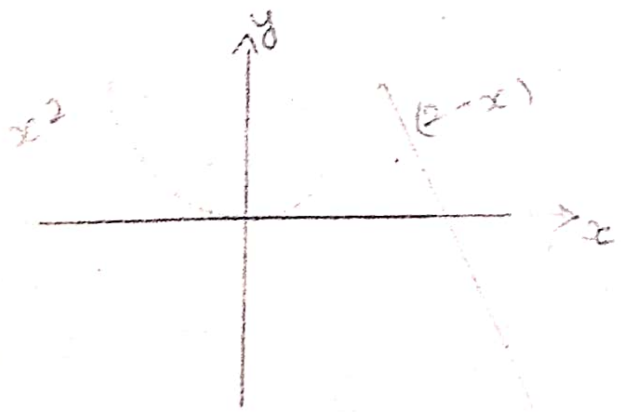
ii) $A^2 = 9, 16, 25$

i) $x_1^2 + 4x_1x_2 + 6x_1x_3 - 2x_2^2 + 8x_2x_3 - 3x_3^2$

i) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$

i) $\lambda_1 = 5$, $\lambda_2 = -2$, $\lambda_3 = 2$

ii) The positive point 5, 2



5. ~~i) $(-\infty,-3)\cup(-3,2)\cup(2,\infty)$~~

1. i) $\tan\sqrt{x} = \sec^2\sqrt{x}$

ii) $\sqrt{x} = 1/2\sqrt{x}$

iii) $\dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

1. i). $\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix}$

. i) $\dfrac{dz}{dt} = 4t^3 + 8a^2t$

. i) $\dfrac{\partial u}{\partial x} = 2x+2y$

ii) $\dfrac{\partial u}{\partial y} = 2y+2x$

iii) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$

iv) $2x^2+2y^2+4xy$

PART-B

16 Marks ::

i) $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

ii) $\boxed{S_1 = 11}$, $\boxed{S_2 = 36}$, $\boxed{S_3 = 36}$

iii) The eigen values are $\lambda = 2,3,6$

iv) The eigen vectors are6 $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

v) The eigen Vectors are $\lambda = 2$ is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

vi) The eigen vector of $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

vii) Rank = 3 , signature = 3 , Index = 3.

I). i) $S_1 = 6$, $S_2 = 11$ $S_3 = 6$

ii) The eigen values are $\lambda = 2, 1, 3$

ii) The eigen vectors of $\lambda = 2$ is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

iii) The eigen vectors of $\lambda = 1$ is $\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$

iv) The eigen vectors of $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

i. $S_1 = 4$, $S_2 = -1$, $S_3 = -4$

ii. $A^2 = \begin{bmatrix} 16 & 18 & 18 \\ 5 & 7 & 6 \\ -5 & -6 & -5 \end{bmatrix}$

iii. $A^3 = \begin{bmatrix} 64 & 78 & 78 \\ 21 & 27 & 26 \\ -21 & -28 & -27 \end{bmatrix}$

iv. $A^{-1} = \begin{bmatrix} 33 & 42 & 42 \\ 9 & 20 & 14 \\ -9 & -22 & -16 \end{bmatrix}$

$S_1 = 3$, $S_2 = -10$, $S_3 = -42$

. $A^2 = \begin{bmatrix} 14 & 1 & 10 \\ 12 & 7 & 2 \\ 2 & 4 & 14 \end{bmatrix}$

i. $A^4 = \begin{bmatrix} 128 & -5 & 154 \\ 204 & 19 & -10 \\ 19 & 64 & 152 \end{bmatrix}$

a

ii. i. $S_1 = 0$, $S_2 = -13$, $S_3 = -12$

ii The eigen values are $\lambda = 3, 1, -4$

iii. The eigen vector of $\lambda = 3$ is $\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

iv. The eigen vector of $\lambda = 1$ is $\begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$

v. The eigen vector of $\lambda = -4$ is $\begin{bmatrix} 1 \\ -3 \\ 13 \end{bmatrix}$

12. b. i. $S_1 = 7$, $S_2 = 9$, $S_3 = 7$

ii. The eigen values are $\lambda = 3, 1, 1$

iii. The eigen vector of $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

iv. The eigen vector of $\lambda = 1$ is $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

v. The eigen vector of $\lambda = 1$ is $\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$

vi. $c = 3x_1^2 + x_2^2 + x_3^2$

b.

i. i. $f'(x) = \frac{1}{2} x^{\frac{1}{2} - 1} - \frac{1}{4} x^{\frac{1}{4} - 1}$

ii. The critical number $x = 1/16$

iii. The first derivative test $f(1/16) = -1/4$

iv. The second derivative test $f(1/16) = 8$.

3.
b.
ii. i. $\frac{dy}{dx} = \frac{x^{3/2}}{\sqrt{x^2+1}\,(3x+2)^5}\left(\frac{3}{2x} - \frac{2x}{x^2+1} - \frac{15(3x+2)}{(3x+2)}\right)$

a
i. i. The critical number $x=-3$, $x=0$

ii The decreasing point $(-\infty,-3)(-3,0)$

iii. The increasing point $(0,\infty)$

iv. The concave upward $(-\infty,0)\cup(2,\infty)$

v. The concave downward $(0,2)$

vi. The inflection point $(0,-16)$

vii. Local minimum at 3

i. i. $a=\frac{5}{2}$

ii. $b=\frac{-1}{2}$

a
i. i. slope $=0$

ii $y=e/2$

i. i. $\frac{d}{dx}\left[(2x+3)^5(x^3-x+1)\right]=5(2x+3)^4\cdot 2(x^3-x+1)+(2x+3)^5(3x^2-1)$

i. $\frac{\partial^2 u}{\partial x^2} = \frac{-2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^2}$

ii. $\frac{\partial^2 u}{\partial y^2} = \frac{-2y^2+2z^2+2x^2}{(x^2+y^2+z^2)^2}$

iii $\frac{\partial^2 u}{\partial z^2} = \frac{-2z^2+2x^2+2y^2}{(x^2+y^2+z^2)^2}$

iv. $\frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2} = \frac{2}{x^2+y^2+z^2}$

ii. i. Maximum at $(-1,0) = 2$

ii Minimum at $(-1,1) = -4$

i. a

i. i. $e^{0} \cos \pi/2 = 0$, $e^{0} \cos \pi/2 = 0$, $-e^{0} \sin \pi/2 = -1$, $e^{x} \cos y = 0$,

$e = -1$, $e = 0$

ii even function $e^{-x} = \frac{1}{e^{x}}$

iii odd function $1/x = -1/x$

ii. i. $F(x, y, z) = f(x, y, z) + \lambda g(x, y, z)$

ii. $x = 6$ $y = 6$ $z = 3$

2. i. i. $\sin u = z$

ii $\frac{x^{2}(1 - y^{3}/x^{3})}{(1 + y/x)}$

iii. By euler's theorem, z is an homogenous function of degree = 2

$$x \frac{dz}{dx} + y \frac{dz}{dy} = nz$$

iv. $x \frac{du}{dx} + y \frac{du}{dy} = 2 \tan u$

v. Hence Proved.

ii i. $f(1, \pi) = 1^{2}\pi + \sin(\pi) + e^{1} = \pi + 0 + e$

SLIPTEST - I

Subject code : MA3151

Date : 7/10/24

MARKS : 20

Department : B.E(CSE)

2M

1) If two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each find the eigen value of A^{-1}

2. If λ is eigen value of matrix A then prove then λ^2 is eigen value of A^2

8M

1) Find the eigen values and eigen vectors of the Matrix

$$B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

2. Verify cayley - Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

SASURIE COLLEGE OF ENGINEERING

DATE : 7/10/24

DEPARTMENT : BE(CSE)

SUBJECT CODE : MA3151

SLIPTEST - I

ANSWER KEY

2M

1. Sum of the eigen value = sum of the main diagonal

$$\lambda_3 = 5$$

The eigen value of A^{-1}

$$\frac{1}{\lambda_3} = \frac{1}{5}$$

2. λ is eigen value of matrix A

$$AX = \lambda X$$

Pre multiple by A

$$A^2X = \lambda^2 X$$

λ^2 is eigen value of A^2 is proved.

8M

1. i) The $S_1 = 7$, $S_2 = 0$, $S_3 = -36$

ii) The eigen values of $\lambda = 2, -3, 6$

iii) The eigen vector of $\lambda = 2$ is $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

iv) The eigen vector of $\lambda = -3$ is $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

v) The eigen vector of $\lambda = 6$ is $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

1. ii) $S_1 = 6$, $S_2 = 5$, $S_3 = -11$

By cayley Hamilton theorem

$$A^3 - 6A^2 + 5A + 11I = 0$$

Slip Test - II

Sub code : MA3151 Date : 19.10.2024

Marks : 20 Department : CSE

2 mark :

1. What is the nature of the quadratic form $x^2+y^2+z^2$ in four variables?

2. Discuss the nature of the quadratic form $2x^2+3y^2+2z^2+2xy$.

16 mark :

3. Reduce the quadratic form into the canonical by using orthogonal transform $x^2+y^2+z^2-2xy-2yz-2zx$, and also find Rank, signature, Index.

Sasurie College of Engineering

Slip Test - II

Department : CSE Date : 19.10.24

Sub code : MA3151

Answer Key

2 mark :

) Positive Semi - definite.

) Positive definite.

16 mark :

) Eigen values $= \lambda = -1, 2, 2$.

Eigen Vectors $= \lambda = -1 \Rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\lambda = 2 \Rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ $\lambda = 2 \Rightarrow \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Canonical form $= \left[-y_1^2 + 2y_2^2 + 2y_3^2\right]$

Nature = Indefinite

Rank = 3

Index $p = 2$

Signature $s = 1$

SLIP TEST - III

SUB CODE : MA3151

DATE : 26.10.2024

MARKS : 20

DEPARTMENT : CSE

2 marks :

1. Find the derivative $y = (x^3 - 1)^{100}$
2. Find the domain and range of $f(x) = \left(\frac{x-4}{x^2-9}\right)$

8 marks

1. (i) Find the value of $\lim_{x \to 0} \frac{\sin x}{x}$

(ii) Evaluate $\lim_{x \to \pi/2} \frac{1+\cos 2x}{(\pi - 2x)^2}$

2. (i) Find an equation of the tangent line to the parabola $y = x^2$ at the point $(1, 1)$

(ii) For what value of the constant "c" is the function "f" continuous on $(-\infty, \infty)$ $f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$

SASURIE COLLEGE OF ENGINEERING

SLIP TEST - III

DEPARTMENT : CSE

DATE: 26.10.2024

SUB CODE : MA3151

ANSWER KEY

2 marks :

1) $100(x^3-1)^{99}(3x^2)$

2) Domain : $(-\infty,-3) \cup (-2,3) \cup (3,\infty)$

Range : $\mathbb{R}$

8 marks

1. (i) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

(ii) $\lim_{x\to \pi/2} \frac{1+\cos 2x}{(\pi-2x)^2} = \frac{1}{2}$

2. (i) $2x - y - 1 = 0$

(ii) $f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$

$c = \frac{2}{3}$

Sasurie College Of Engineering.

Subject code :- MA3151 Date : 20.11.2024

Mark :- 20 Department :- BE-CSE

Slip test -4

2 Marks

1) Find the critical Point of $y = 5x^3 - 6x$

2) State Mean value theorem.

8 Marks

1) Find the local maximum and minimum value of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both the first and second derivatives test.

2) Given the equation $x^2 + xy + y^3 = 1$; to find y''

Answer

2 Marks:

1) **Solution:**

The critical points are,

$x = \sqrt{\frac{2}{5}}$ $\quad x = -\sqrt{\frac{2}{5}}.$

) **Solution:**

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b-a}}$$

8 Marks:

1) **Solution:**

$\therefore$ f is local minimum at $x = \frac{1}{16}$

$\therefore$ f is local minimum value is $-\frac{1}{4}$.

2) **Solution:**

$$\boxed{y'' = 2}$$

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, tirupur 638056

(Approved by AICTE, New Delhi and affiliated to Anna University, Chennai)

Internal Assessment Answer Book

Name	B. Praleena			Year/ Semester/Section		I - B	
Batch No.		Date/Session	12/11/24	Department		BE.CSE	
Course code	MA3151	Course Title	Matrices and Calculus				
Internal Assessment Test		IAT 1 ☑	IAT 2 ☐	IAT 3 ☐		Model ☐	
Name and Signature of the Invigilator with date			G. Jeevanandham. 12/11/24				

Instruction to the Student: Put tick mark to the question attended in the column against question.

Part A			Part B/ Part C					
Q. No.	✓	Marks	Q. NO.	✓	a Marks	✓	b Marks	Total Marks
1	✓	2	11	✓			15	15
2	✓	2	12	✓	8			8
3	✓	2	13					
4	✓	2	14					
5	✓	2	15					
6	✓	2	16					
7			Grand Total					23
8	✓	2	39			G. Jeevanandham 12/11/2024.		
9								
10	✓	2						
Total		16	Grand Total			Name and Signature of the Examiner with date		

To be filled by the examiner							
Course Outcomes	1	2	3	4	5	6	Total
Marks allotted	25	25	-	-	-	-	50
Marks Obtained	25	14					39.

IQAC Audit - Remarks

Good. Need a improvement.

M. Sathya

Name and Signature of the IQAC member

29/30

DATE / /

Internal test - 1

Name : B. Prapeena code : MA3151
Section : B roll no : 24CS072
Department : BE CSE Date : 12.11.24
Subject : Matrices and calculus

PART - A

1. Eigen values $= +2, -1, -3$ of the matrix A. Find Eigen value of $A^2 - 2I$

Solution :-

Eigen value of $A^2 - 2I$ B.Prapeena

$= 4-2, 1-2, 9-2$

$= (2, -1, 7)$ is eigen value of $A^2 - 2I$.

2. 2, 3 are the two eigen values of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$ Find value of b.

Solution:-

Given matrix $= \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$

$\lambda 1 = 2, \lambda 2 = 3, \lambda 3 = ?$

sum of the eigen values = sum of the diagonal element

$\lambda 1 + \lambda 2 + \lambda 3 = 2+2+2$

$2 + 3 + \lambda 3 = 6$

$5 + \lambda 3 = 6$

$\lambda 3 = 6 - 5 = \boxed{\lambda 3 = 1}$

matrix $= \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{bmatrix}$

$$\lambda_1\lambda_2\lambda_3 = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$$

$$\lambda_1\lambda_2\lambda_3 = 2(4-0) + 0 + 1(0-2b)$$

$$2.3.1 = 8-2b$$

$$6 = 8-2b$$

$$2b = 8-6$$

$$2b = 2$$

$$\boxed{b=1}$$

given matrix $= \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

3. matrix corresponding to the quadratic form.

$$2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1.$$

$$A = \begin{array}{c|ccc|} & x_1 & x_2 & x_3 \\ x_1 & 2 & 2 & 1 \\ x_2 & 2 & 5 & 0 \\ x_3 & 1 & 0 & 0 \end{array}$$

4. Solution:-

Eigen values are $\lambda_1, \lambda_2, \lambda_3$,

Sum of the eigen value = Trace of A.

Trace of A - Sum of two eigen values

$$\lambda_1 + \lambda_2 + \lambda_3 = \lambda_1 + \lambda_2$$

$$\lambda_3 = 0$$

product of $|A|$

$\lambda_3 = 0$.

$\lambda_1 \lambda_2 \lambda_3 = 0$

$\therefore \lambda_3 = 0 \quad \lambda_1 \lambda_2 (0) = 0$

So The sum of two eigen value and trace of a 3x3 matrix A are equal.

---

5. Eigen values. $= \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

CE eqn $= |A - \lambda I| = 0$

ie $= \lambda^3 - S_1 \lambda^2 + S_2 \lambda - S_3 = 0$

$$\begin{array}{r} 14 \times 3 \\ \hline 42 \end{array}$$

$S_1 = 1 + 5 + 1 = 7$

$S_2 = (5-1) + (1-9) + (5-1)$

$= 4 - 8 + 4$

$= 0$

$S_3 = 1(5-1) - 1(1-3) + 3(1-15)$

$= 1(4) - 1(-2) + 3(-14)$

$= 4 + 2 - 42$

$= 6 - 42$

$= -36$

$$\begin{array}{r} -42 \\ 6 \\ \hline -36 \end{array}$$

CE are $\Rightarrow \lambda^3 - 7\lambda^2 + 0\lambda + 36 = 0$

$$\begin{array}{r|rrrr} -2 & 1 & -7 & 0 & 36 \\ & 0 & -2 & 18 & -36 \\ \hline & 1 & -9 & 18 & \boxed{0} \end{array}$$

$\lambda = -2, \ \lambda^2 - 9\lambda + 18 = 0$

$\lambda = -2, \ (\lambda - 6)(\lambda - 3) = 0$

$\lambda = -2, \ \lambda = 6, \ \lambda = 3$

18

-6 -3

-9

DATE / /

6. derivative $y = (x^3-1)^{100}$

$$\frac{dy}{dx} = 100(x^3-1)^{99} \cdot 3x^2$$
$$= 300x^2(x^3-1)^{99}$$

8. Critical points : $y = 5x^3 - 6x$.

$$\frac{dy}{dx} = 5 \cdot (3x)^2 - 6$$
$$= 15x^2 - 6$$
$$= 15x^2 = 6$$
$$x^2 = \frac{\cancel{6}\,2}{\cancel{15}\,5}$$
$$x^2 = \frac{2}{5}$$
$$\boxed{x = \pm\sqrt{2/5}.}$$

10. Evaluate limits for $\lim_{x \to -2} \frac{x+2}{x^3+8}$

$$f(x) = \frac{x+2}{x^3+8}$$
$$= \frac{-2+2}{(-2)^3+8} = \frac{0}{-8+8} = \frac{0}{0}.$$

L. Hospital rule.

$$f'(x) = \frac{1+0}{3x^2+0} = \frac{1}{3x^2} = \frac{1}{3(-2)}$$

$$\Rightarrow \lim_{x \to -2} \frac{1}{3x^2}$$

$$= \frac{1}{3(-2)^2} = \frac{1}{3(4)} = \frac{1}{12}$$

PART - B.

11. b.

$$6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$$

matrix form.

let, $A = \begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 6 & -2 & 2 \\ x_2 & -2 & 3 & -1 \\ x_3 & 2 & -1 & 3 \end{array}$

characteristic equation $= |A - \lambda I| = 0$

(i.e) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$.

$S_1 = 6+3+3 = 12$

$S_2 = (9-1)+(18-4)+(18-4)$

$= 8+14+14$

$= 36$

$S_3 = 6(9-1)+2(-6+2)+2(2-6)$

$= 6(8)+2(-4)+2(-4)$

$= 48-8-8$

$= 48-16$

$= 32$

28
8
36

48
16
32

The CE are,

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0.$$

$$\lambda^3 - 12\lambda^2 + 36\lambda - 32 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 36 & -32 \\ & 0 & 2 & -20 & 32 \\ \hline & 1 & -10 & 16 & \boxed{0} \end{array}$$

$\lambda = 2,\ \lambda^2 - 10\lambda + 16 = 0.$

$\lambda = 2,\ \lambda^2 - 10\lambda + 16 = 0$

$$\begin{array}{ccc} & 16 & \\ -8 & & -2 \\ & -10 & \end{array}$$

$\lambda = 2,\ (\lambda - 8)(\lambda - 2) = 0$

$\lambda = 2,\ \lambda = 8,\ \lambda = 2$

The eigen Values are $\lambda = 2, \lambda = 2,$
$\lambda = 8.$

To find eign Vector:-

$(A - \lambda I)X = 0.$

case (i) if $\lambda = 8.$

$(A - \lambda I)X = 0$
$(A - 8I)X = 0.$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6-8 & -2 & 2 \\ -2 & 3-8 & -1 \\ 2 & -1 & 3-8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

equations are,

$-2x_1 - 2x_2 + 2x_3 = 0$ — (1) *

$-2x_1 - 5x_2 - x_3 = 0$ — (2) *

$2x_1 - x_2 - 5x_3 = 0$ — (3)

x_1	x_2	x_3	
-2	2	-2	-2
-5	-1	-2	-5

$$\frac{x_1}{(2+10)} = \frac{x_2}{(-4-2)} = \frac{x_3}{(10-4)} \Rightarrow \frac{x_1}{12} = \frac{x_2}{-6} = \frac{x_3}{6}$$

6 *

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{-1} = \frac{x_3}{1}$$

The eign vector of $\lambda = 8$ is

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

if $\lambda = 2$ is.

$$(A - \lambda I)x = 0$$

$$(A - 2I)x = 0$$

$$(A - 2I)x = 0.$$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6-2 & -2 & 2 \\ -2 & 3-2 & -1 \\ 2 & -1 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

DATE / /

$4x_1 - 2x_2 + 2x_3 = 0$ — ①

$-2x_1 + x_2 - x_3 = 0$ — ②

$2x_1 - x_2 + x_3 = 0$ — ③

case (ii)

$x_1 = 1 \quad x_2 = 2.$

③ => $2x_1 - x_2 + x_3 = 0.$

$2(1) - 2 + x_3 = 0$

$2 - 2 + x_3 = 0$

$0 + x_3 = 0$

$\boxed{x_3 = 0}$

The eign vector of $\lambda = 2$ is $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

case (iii)

$x_1 = 2 \quad x_2 = 3$

③ => $2x_1 - x_2 + x_3 = 0$

$2(2) - 3 + x_3 = 0$

$4 - 3 + x_3 = 0$

$1 + x_3 = 0$

$\boxed{x_3 = -1}$

The eign vector of $\lambda = 2$ is $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$

The eign vector matrix :

$\sqrt{9+1+4}$ $\sqrt{9}=3$

$$= \begin{bmatrix} 2 & 1 & 2 \\ -1 & 2 & 3 \\ 1 & 0 & -1 \end{bmatrix}$$

9+4+1
13

To find N

$$N = \begin{bmatrix} 2/\sqrt{2^2+1^2+2^2} & \frac{1}{\sqrt{2^2+1^2+2^2}} & \frac{2}{\sqrt{2^2+1^2+2^2}} \\ -1/\sqrt{(-1)^2+2^2+3^2} & \frac{2}{\sqrt{(-1)^2+2^2+3^2}} & 3/\sqrt{(1)^2+2^2+3^2} \\ 1/\sqrt{1^2+0^2+(-1)^2} & 0/\sqrt{1^2+0^2+(-1)^2} & -1/\sqrt{1^2+0^2+(-1)^2} \end{bmatrix}$$

$$N = \begin{bmatrix} 2/\sqrt{9} & \frac{1}{\sqrt{9}} & \frac{2}{\sqrt{9}} \\ -1/\sqrt{14} & 2/\sqrt{14} & 3/\sqrt{14} \\ 1/\sqrt{2} & 0/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -1/\sqrt{14} & 2/\sqrt{14} & 3/\sqrt{14} \\ 1/\sqrt{2} & 0/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$N^T = \begin{bmatrix} 2/\sqrt{9} & -1/\sqrt{14} & 1/\sqrt{2} \\ 1/\sqrt{9} & 2/\sqrt{14} & 0 \\ 2/\sqrt{9} & 3/\sqrt{14} & -1/\sqrt{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2/3 & -1/\sqrt{14} & 1/\sqrt{2} \\ 1/3 & 2/\sqrt{14} & 0 \\ 2/3 & 3/\sqrt{14} & -1/\sqrt{2} \end{bmatrix}$$

$D = N^T A N$

$$= \begin{bmatrix} 2/3 & -1/\sqrt{14} & 1/\sqrt{2} \\ 1/3 & 2/\sqrt{14} & 0 \\ 2/3 & 3/\sqrt{14} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -1/\sqrt{14} & 2/\sqrt{14} & 3/\sqrt{14} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

To find canonical form.

$$C = y^T D y \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1}$$

$$C = [y_1 \ y_2 \ y_3]_{1\times3} \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{(3\times3)} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1}$$

$$= [8y_1 \ 2y_2 \ 2y_3]_{1\times3} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}_{3\times1}$$

$$= [8y_1^2 + 2y_2^2 + 2y_3^2]_{1\times1}$$

To find Nature of the matrix.

All the eign value ~~of~~ are positive $\lambda = 8, \lambda = 2, \lambda = 2$.

So, its possitive defintie.

$$D = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

index (P) = number of possitive value in diagonal.

$$P = 3$$

Rank (r) = number of non zero value in diagonal

$$r = 3$$

signature $= 2p - r$

$$= 2(3) - 3$$

$$= 6 - 3$$

$$= 3$$

b)

(i) Determine whether $f'(0)$ exist or not for the given function.

$$f(x) = \begin{cases} x \sin(1/x) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

Solution.

$$f(x) = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{h \sin(1/h) - 0}{h}$$

$$= \lim_{h \to 0} \sin 1/h - 0$$

$$= \lim_{h \to 0} \sin 1/0$$

$$f(x) = \infty$$

The given function are not exist.
$f'(0)$ is not exist.

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, Tirupur 638056

(Approved by AICTE, New Delhi and affiliated to Anna University, Chennai)

Internal Assessment Answer Book

Name	S. Sutthishna			Year/ Semester/Section	I - 'B'		
Batch No.		Date/Session	27/11/24	Department	ESE		
Course code	MA3151	Course Title	Matrices and calculus				
Internal Assessment Test		IAT 1 ☐	IAT 2 ☑	IAT 3 ☐		Model ☐	
Name and Signature of the Invigilator with date			V.DEEPAN 27/11/24				

Instruction to the Student: Put tick mark to the question attended in the column against question.

Part A			Part B/ Part C					
Q. No.	✓	Marks	Q. NO.	✓	a Marks	✓	b Marks	Total Marks
1			11	✓	7			7
2	✓	2	12	✓			13	13
3	✓	2	13					
4	✓	2	14					
5			15					
6	✓	2	16					
7	✓	1					Grand Total	20
8			29				G. Jeevanantham 27/11/24	
9								
10	✓	0						
Total		9	Grand Total				Name and Signature of the Examiner with date	

To be filled by the examiner

Course Outcomes	1	2	3	4	5	6	Total
Marks allotted	–	25	25	–	–	–	50
Marks Obtained	–	13	16	–	–	–	29

IQAC Audit - Remarks	
Written Practice is Must needed.	M. Sathya [M. Sathya] Name and Signature of the IQAC member

CHOLA
DATE: __/__/__
PAGES

Name: S. Suthishna Sub: matrices and calculus.
Class: CSE
Sec: "B" Date: 27/11/2024
Roll no: 24CS102 Sub code: MA3151
Exam: Internal Exam II

29/50

part - A

2 mark:

2. Prove that $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

Soul:

Given $x \to 0 \; \frac{|x|}{x}$

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\lim_{x \to 0^+} = \lim_{x \to 0^+} = \frac{x}{x} = 1$$

$$\lim_{x \to 0^-} = \lim \frac{|x|}{x} = \lim_{x \to 0^-} = \frac{-x}{x} = -1$$

$\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

4. Evaluate the limit $\lim_{x \to 1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

Solu:

Given $\lim_{x \to 1} \frac{x^2 - 4x}{x^2 - 3x - 4}$

$$\lim_{x \to 1} = \frac{1^2 - 4(1)}{(1)^2 - 3(1) - 4}$$

$$= \frac{1-4}{1-3-4}$$

$$= \frac{-3}{-6}$$

$$= \frac{\cancel{3}\,1}{\cancel{6}\,2}$$

$$= \frac{1}{2}$$

7. If $u = x^3 + y^3$ when $x = \cos t$ $y = b \sin t$ the find $\frac{du}{dt}$.

Solu:

Given $u = x^3 + y^3$, $x = \cos t$, $y = b \sin t$

$u = x^3 + y^3$, $x = \cos t$, $y = b \sin t$

$\frac{du}{dx} = 3x^2 + y^2$	$\frac{dx}{dt} = a \sin t$	$\frac{du}{dt} = b \cos t$
$\frac{du}{dy} = x^3 + 3y^2$		

$$\frac{du}{dt} = \frac{du}{dx} \cdot \frac{dx}{dt} + \frac{du}{dy} \cdot \frac{dy}{dt}$$

part - B

11a) ii) Find the local maximum and local minimum values of $f(x) = \sqrt{x} + 4\sqrt{x}$ using first and second derivatives test.

Soln:

Given $f(x) = \sqrt{x} + 4\sqrt{x}$

$= x^{1/2} - x^{1/4}$

$f'(x) = 1/2\, x^{1/2 - 1} - 1/4\, x^{-1/4 - 1}$

$= 1/2\, x^{-1/2} - 1/4\, x^{-3/4}$

$= 1/2\, x^{-3/4} (2x^{1/4} - 1)$

$f'(x) = 1/4\, x^{-3/4} (2x^{1/4} - 1)$

To find critical numbers

$f'(x) = 0$

$1/4\, x^{-3/4} (2x^{1/4} - 1) = 0$

$2x^{1/4} - 1 = 0$

$2x^{1/4} = 1$

$x^{1/4} = 1/2$

$x = (1/2)^4$

$x = 1/16$

The critical number is $x = 1/16$

First Derivative test:

$f(1/16) = \sqrt{1/16} - 4\sqrt{1/16}$

$= (1/4)^{2 \cdot 1/2} - (1/2)^{4 \cdot 1/4}$

$= 1/4 - (1/2) = \frac{1-2}{4}$ $(f\,1/16) = \frac{-1}{4} < 0$

$\therefore f$ is local minimum number $= x = 1/16$

$\therefore f$ is local minmum ~~number~~ value are $f(1/16) = -1/4$

Second Derivative test:

$f''(x) = (1/2)(-1/2)x^{1/2 - 1}) - 1/4(-3/4)x$

$= -1/4\, x^{-3/2} + \frac{3}{16} x^{-7/4}$

$= 1/4 x^{-7/4} \left(\frac{-4x^{1/4} + 3}{4}\right)$

$= 1/16 x^{-7/4}(-4x^{1/4} + 3)$

$= 1/16 x^{-7/4}(-4x^{1/9} + 3)$

$f''(1/16) = 1/16(1/16)^{-7/4}$

$\left[-4(1/16)^{1/9} + 3\right]$

$$= \frac{1}{16}\left(\frac{1}{2^4}\right)^{-7/4}\left[-4\left(\frac{1}{2^4}\right)^{1/4}+3\right]$$

$$= \left(\frac{1}{16}\right)\left(\frac{1}{2^{-7}}\right)\left[-4\left(\frac{1}{2}\right)+3\right)$$

$$= \frac{1}{16}(2^7)(-2+3)$$

$$= \frac{1}{16}(128)(1)$$

$$= 8 > 0$$

9)

12b. Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm.

Solu: $g(x,y,z) = xy + 2yz + 2zx = 108$

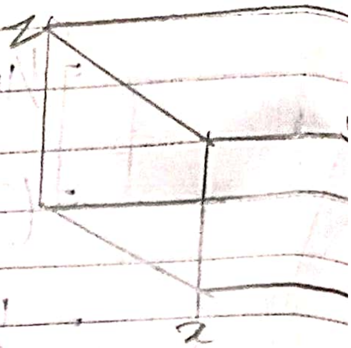
$f(x,y,z) = xyz$

$F(x,y,z) = xyz + \lambda(xy + 2yz + 2zx - 108)$

$$\frac{\partial F}{\partial x} = yz + \lambda(y+2z) \longrightarrow ①$$

$$\frac{\partial F}{\partial y} = xz + \lambda(x+2z) \longrightarrow ②$$

$$\frac{\partial F}{\partial z} = xy + (2x+2y) \longrightarrow ③$$



$\frac{\partial F}{\partial x} = 0 \qquad \frac{\partial f}{\partial y} = 0$

$yz + \lambda(y+2x) = 0 \longrightarrow (1)$

$xz + \lambda(x+2z) = 0 \longrightarrow (2)$

From (1) & (2)

$(1) \times x \Rightarrow xyz + \lambda xy + \lambda 2xz = 0$

$(2) \times y \Rightarrow xyz + \lambda xy + \lambda 2yz = 0$

$\lambda 2xz - \lambda 2yz = 0 \longrightarrow (3)$

$\lambda 2xz - \lambda 2yz = 0$

$\lambda 2z(x-y) = 0$

$x - y = 0$

$\boxed{x = y}$

From equation (2) & equation (3)

$(2) \times y \Longrightarrow xyz + \lambda xy + \lambda 2yz = 0$

$(3) \times z \Longrightarrow xyz + \lambda 2xz + \lambda 2zy = 0$

$\lambda xy - \lambda 2xz = 0$

$\lambda x(y - 2z) = 0$

$y - 2z = 0$

$y = 2z$

$\boxed{y/2 = z}$

$S = xy + 2yz + 2zx - 108$

$\Rightarrow x^2 + 2x(y/2) + 2(y/2)\,y = 108$

$\Rightarrow x^2 + x^2 + x^2 = 108$

$\Rightarrow 3x^2 = 108$

$x^2 = \frac{108}{3}$

$x^2 = 36$

$\boxed{x = 6}$ //

ii) 32 cc $\quad (S) = xy + 2yz + 2zx$

Volume $= xyz = 32$

Solu: $\boxed{V = xyz - 32}$

$F(x,y,z) = xy + 2yz + 2zx + \lambda(xyz - 32)$

$\frac{dF}{dx} = y + 2z + \lambda yz$

$\frac{dF}{dy} = x + 2z + \lambda xz$

$\frac{dF}{dz} = 2y + 2x + \lambda xy$

$\frac{dF}{d\lambda} = xyz - 32$

$\frac{dF}{dx} = 0,$

$$y+2z+\lambda yz=0$$

$$y+2z=-\lambda yz$$

$$\frac{y+2z}{yz}=-\lambda$$

$$\frac{y}{yz}+\frac{2z}{yz}=-\lambda$$

$$\frac{\partial F}{\partial y}=0$$

$$x+2z+\lambda xz=0$$

$$x+2z=-\lambda xz$$

$$\frac{x+2z}{xz}=-\lambda$$

$$\frac{x}{xz}+\frac{2z}{xz}=-\lambda$$

$$\frac{1}{z}+\frac{2}{y}=-\lambda \longrightarrow (1)$$

$$\frac{1}{z}+\frac{2}{x}=-\lambda \longrightarrow (2)$$

$$\frac{\partial F}{\partial z}=0,$$

$$2y+2x+\lambda xy$$

$$2y+2x=-\lambda xy$$

$$\frac{2y+2x}{xy}=-\lambda$$

$$\frac{2y}{xy}+\frac{2x}{xy}=-\lambda$$

$$\frac{2}{x}+\frac{2}{y}=-\lambda \rightarrow (3)$$

$$\frac{1}{z}+\frac{2}{y}=\frac{1}{z}+\frac{2}{x}$$

$$\frac{2}{y}=\frac{2}{x}$$

$$\frac{1}{y}=\frac{1}{x}$$

$$\boxed{x=y} \rightarrow (4)$$

$$\frac{1}{z}+\frac{2}{x}=\frac{2}{x}+\frac{2}{y}$$

$$\frac{1}{z}=\frac{2}{y}$$

$$\boxed{y=2z} \rightarrow (5)$$

From equation (4) & (5)

$$x=y=2z$$

$$V=xyz=32$$

$$xyz=32$$

$$(2z)(2z)(z)=32$$

$$4z^3=32$$

$$z^3=\frac{32}{4}$$

$$z^3=8$$

$$z=2$$

$$\boxed{x=4}$$

$$\boxed{y=4}$$

The answer is equal.

6. write Euler's theorem on homogeneous function:

Solu:

If u is an homogeneous function degree 'n' in the variable 'x' and 'y'

Then,

$$\boxed{x\frac{du}{dx}+y\frac{du}{dy}=nu.}$$

10. Find $\frac{dy}{dx} = x3+y3 = 3axy.$

Solu. Given $\frac{dy}{dx} = x3+y3 = 3axy$

$$\frac{dy x^3}{dx} + y^3\frac{dy}{dx} = 3axy\frac{dy}{dx}$$

$= 2x^2+y^2 = 3ay$

$= 2x^2+y = 3a$

$a = \frac{2x^2+y}{3}$ //

3. Find the Slope of the circle $x^2+y^2=25$ at $(3,-4)$

Solu: Given $= x^2+y^2=25$

$$x^2+y^2=25$$

$$\frac{dy}{dx}=x^2+y^2=25$$

$$\frac{d}{dx}(x^2+y^2)=\frac{d}{dx}25$$

$$2x+2yy'=0$$

$$y'=\frac{-2x}{2y}$$

$$y'=\frac{-x}{y}$$

$$\left(\frac{dy}{dx}\right)_{(3,-4)}=\frac{-x}{y}$$

$$=\frac{+3}{+4}$$

$$=\frac{3}{4}$$

MA3151

Name : K. Poorna

Class : Sec 'B'

Department : BE.CSE

Roll No : 24CS074

Subject : Matrices and Calculus

Slip test No : 01

Date : 04.10.2024

18/20

K. Poorna

1).

Given : $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda_1 = 1, \lambda_2 = 1$

To Find $\lambda_3 = ?$

$\lambda_1 + \lambda_2 + \lambda_3 = 2 + 3 + 2$

$1 + 1 + \lambda_3 = 7$

$2 + \lambda_3 = 7$

$\lambda_3 = 7 - 2$

$\boxed{\lambda_3 = 5}$

$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 5$

$\lambda_1, \lambda_2, \lambda_3$ is an eigen value of A.

$1/\lambda_1, 1/\lambda_2, 1/\lambda_3$ is a eigen value of A^{-1}

To Find the A^{-1}

TNPL

$$\frac{1}{\lambda_1} = \frac{1}{1} = 1$$

$$\frac{1}{\lambda_2} = \frac{1}{1} = 1$$

$$\frac{1}{\lambda_3} = \frac{1}{5}$$

21. given: λ is eigen value of matrix of A

$$\cancel{Ax}\ (A - \lambda I)x = 0$$

$$Ax - \lambda I x = 0$$

$$Ax - \lambda x = 0$$

$$Ax = \lambda x \longrightarrow (1)$$

pre multiple A

$$A(Ax) = A(\lambda x)$$

$$A^2 x = A \lambda x$$

from equation (1)

$$A^2 x = \lambda(\lambda x)$$

$$A^2 x = \lambda^2 x.$$

The λ^2 is eigen value of A^2

Hens proved.//

3)

given: $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

(or)

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1 + 5 + 1$

$S_1 = 7$

$S_2 = \begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix}$

$= (5-1) + (1-9) + (5-1)$

$= 4 - 8 + 4$

$S_2 = 0$

$S_3 = |A|$

$= 1(5-1) - 1(1-3) + 3(1-15)$

$= 1(4) - 1(-2) + 3(-14)$

$= 4 + 2 - 42$

$= -36$

$\lambda^3 - 7\lambda^2 + 0\lambda + 36 = 0$

$$\begin{array}{r|rrrr} 6 & 1 & -7 & 0 & 36 \\ & 0 & 6 & -6 & -36 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$$y=6,\quad y^2-y-6=0$$

$$-6 \;\to\; -3,\ 2$$

$$y=6,\quad (y+2)(y-3)=0$$

$$y=6,-2,3$$

To Find the eigen ~~values~~ vector

case (i)

$y=6$ If $(A-\lambda I)X=0$

$$\begin{bmatrix} 1-6 & 1 & 3 \\ 1 & 5-6 & 1 \\ 3 & 1 & 1-6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x_1+x_2+3x_3=0$

$x_1-x_2+x_3=0$

$3x_1+x_2-5x_3=0$

x_1		x_2	x_3
-1	1	1	-1
1	-5	3	1

$$\frac{x_1}{5-1}=\frac{x_2}{3+5}=\frac{x_3}{1+3}$$

$$\frac{x_1}{4}=\frac{x_2}{8}=\frac{x_3}{4} \Rightarrow \frac{x_1}{1}=\frac{x_2}{2}=\frac{x_3}{1}$$

$y=6$ is eigen ~~value~~ vector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

case (ii) $y=-2$ $(A+2I)X=0$

$$\begin{bmatrix} 1+2 & 1 & 3 \\ 1 & 5+2 & 1 \\ 3 & 1 & 1+2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + x_2 + 3x_3 = 0$

$1x_1 + 7x_2 + 1x_3 = 0$

$3x_1 + x_2 + 3x_3 = 0$

x_1		x_2	x_3
7	1	1	7
1	3	3	1

$$\frac{x_1}{21-1} = \frac{x_2}{3-3} = \frac{x_3}{1-21} \Rightarrow \frac{x_1}{20} = \frac{x_2}{0} = \frac{x_3}{-20}$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1}$$

$\lambda = -2$ is eigen ~~value~~ vector $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

case(iii) $\lambda = 3 \Rightarrow (A - 3I)x = 0$

$$\begin{bmatrix} 1-3 & 1 & 3 \\ 1 & 5-3 & 1 \\ 3 & 1 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & +2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + 3x_3 = 0$

$x_1 + 2x_2 + x_3 = 0$

$3x_1 + x_2 - 2x_3 = 0$

x_1		x_2	x_3
2	1	1	2
1	-2	3	1

$$\frac{x_1}{-4-1} = \frac{x_2}{3+2} = \frac{x_3}{1-6} \Rightarrow \frac{x_1}{-5} = \frac{x_2}{5} = \frac{x_3}{-5}$$

$$\frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{-1}$$

$\lambda = 3$ is eigen values vector $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

4)

given : $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$|A - \lambda I| = 0$ (or) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1 + 2 + 3$
$= 6$

$S_2 = \begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$= (6+3) + (3-2) + (2-1)$

$6\lambda^2$ $= 3 + 1 + 2$

$S_2 = 5$

$S_3 = |A|$

$= 1(6-3) - 1(3+6) + 1(-1-4)$

$= 1(3) - 1(9) + 1(-5)$

$= 3 - 9 - 5$

$S_3 = -11$

$$\lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0$$

$$A^3 - 6A^2 + 5A + 11I = 0$$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2+2+3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$A^3 = A^2 A$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16-14 & -3-24-42 \\ 7-3+28 & 7-6+14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$6A^2 = \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix}$$

$$5A = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix}$$

$$11I = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

verify?

$$A^3 - 6A^2 + 5A + 11I = 0$$

$\times A^{-1} = 1$

$$A^{-1}A^3 - 6A^{-1}A^2 + 5A^{-1}A + 11A^{-1}I = 0$$

$$A^2 - 6A + 5I + 11A^{-1} = 0$$

$$11A^{-1} = -A^2 + 6A - 5I$$

$\cancel{11A^{-1}}$

$$-A^2 = \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix}$$

$$6A = \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix}$$

$$5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = -A^2 + 6A - 5I$$

$$= \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix} + \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} -4+6-5 & -2+6-0 & -1+6-0 \\ 3+6-0 & -8+12-0 & 14+18+0 \\ -7+12-0 & 3-6-0 & -14+18-5 \end{bmatrix}$$

~~A~~

$$A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

-1

SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, tirupur 638056

(Approved by AICTE, New Delhi and affiliated to Anna University, Chennai)

Internal Assessment Answer Book

Name	P.V.K Sarmitha			Year/ Semester/Section	I-I-'B'
Batch No.		Date/Session	11/12/24	Department	CSE
Course code	MA3151	Course Title	MATRICES AND CALCULUS		
Internal Assessment Test		IAT 1 ☐	IAT 2 ☐	IAT 3 ☐	Model ☑
Name and Signature of the Invigilator with date			V. Papamy 11/12/24		

Instruction to the Student: Put tick mark to the question attended in the column against question.

Part A			Part B/ Part C					
Q. No.	✓	Marks	Q. NO.	✓	a Marks	✓	b Marks	Total Marks
1	✓	0	11	✓	16			16
2	✓	2	12	✓	16			16
3	✓	0	13	✓	15			15
4	✓	2	14	✓	16			16
5	✓	0	15	✓	16			16
6	✓	2	16					
7	✓	2					Grand Total	79
8	✓	2	93			G. Jeevanantham 14/12/2024		
9	✓	2						
10	✓	2						
Total		14	Grand Total			Name and Signature of the Examiner with date		

To be filled by the examiner

Course Outcomes	1	2	3	4	5	6	Total
Marks allotted	40	22	38	–	–	–	100
Marks Obtained	36	19	38	–	–	–	93

IQAC Audit - Remarks

Good. Need to Maintance this result.

Name and Signature of the IQAC member

DATE :

MODEL EXAM - 1

Name : P.V.K. Sarmitha   Sub code : MA3151

Reg No : 732424104088   Date : 11/12/24

Subject : Matrices and calculus   Department : B.E computer science

91

93/100

PART - A

1. $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$ find A^{-1} and A^2

$A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$

$A^2 = A \times A$

$= \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9+0+0 & 0+0+0 & 0+0+0 \\ 24+32+0 & 0+16+0 & 0+0+0 \\ 18+16+30 & 0+8+10 & 0+0+25 \end{bmatrix}$

$\begin{array}{r} 18 \\ 16 \\ \hline 34 \\ 30 \\ \hline 64 \end{array}$

$= \begin{bmatrix} 9 & 0 & 0 \\ 56 & 16 & 0 \\ 64 & 18 & 25 \end{bmatrix}$

$A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$

The characteristic eqn is $|A - \lambda I| = 0$

(ie) $\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$

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$S_1 = 3+4+5 = 12$

$S_2 = \begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 6 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 8 & 4 \end{vmatrix}$

$= (20-0)+(15-0)+(12-0)$

$= 20+15+12 = 47$

$S_3 = |A|$

$= 3(20-0) - 0 + 0$

$= 3(20) = 60$

$$\begin{array}{c|cccc} 1 & 1 & -12 & 47 & -60 \\ & 0 & 1 & -11 & 36 \\ \hline & 1 & -11 & 36 & \boxed{-24} \end{array}$$

$$\begin{array}{c|cccc} 5 & 1 & -12 & 47 & -60 \\ & 0 & 5 & -35 & 60 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

The C.E is

$\lambda^3 - 12\lambda^2 + 47\lambda - 60 = 0$

$\lambda = 1, \; \lambda^2 - 7\lambda + 12 = 0$

$\lambda = 1, \; (\lambda-4)(\lambda-3)$

$\lambda = 1, \; \lambda = 3, \; \lambda = 4$

$A^{-1} = \frac{1}{|A|}(\mathrm{adj}\, A)$

$= \frac{1}{60}\begin{bmatrix} (20-0) & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 12 \end{bmatrix}$

$A^{-1} = \frac{1}{60}\begin{bmatrix} 20 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 12 \end{bmatrix}$ //

2. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & -4 \\ 3 & -4 & -3 \end{bmatrix}$ Write the quadratic form.

Soln.,

The quadratic form of the matrix,

$x^2 - 2y^2 - 3z^2 + 4xy + 6xz - 8yz$

20
15
12
—
47

12
-4 -3
-7

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4. $x^2 - y^2 + 4z^2 + 4xy + 2yz + 6xz$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

The C.E eqn is $|A - \lambda I| = 0$

(ie) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1 - 1 + 4 = 4$

$$S_2 = \begin{vmatrix} -1 & 1 \\ 1 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$$

$= (-4-1) + (4-9) + (-1-4)$

$= -5 - 5 - 5$

$= -15$

$S_3 = |A| = 1(-4-1) - 2(8-3) + 3(+2+3)$

$= 1(-5) - 2(5) + 3(5)$

$= -5 - 10 + 15$

$= 0$

The C.E is

$\lambda^3 -$

$D_1 = |1| = 1$

$$D_2 = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = (-1-4) = -5$$

$$D_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 1(-4-1) - 2(8-3) + 3(5)$$

$= 1(-5) - 2(5) + 15$

$= -5 - 10 + 15$

$= 0$

$D_1 = 1 > 0$, $D_2 = -5 < 0$ $D_3 = 0$

$\therefore$ It is indefinite

Hence Proved.

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5. Sketch the graph $f(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 0 \\ 2-x & \text{if } 0 < x \le 2 \end{cases}$

$x^2 \Rightarrow x = -2$

$x^2 = 4 \Rightarrow (-2, 4)$

$x = -1$

$x^2 = 1 \Rightarrow (-1, 1)$

$x = 0$

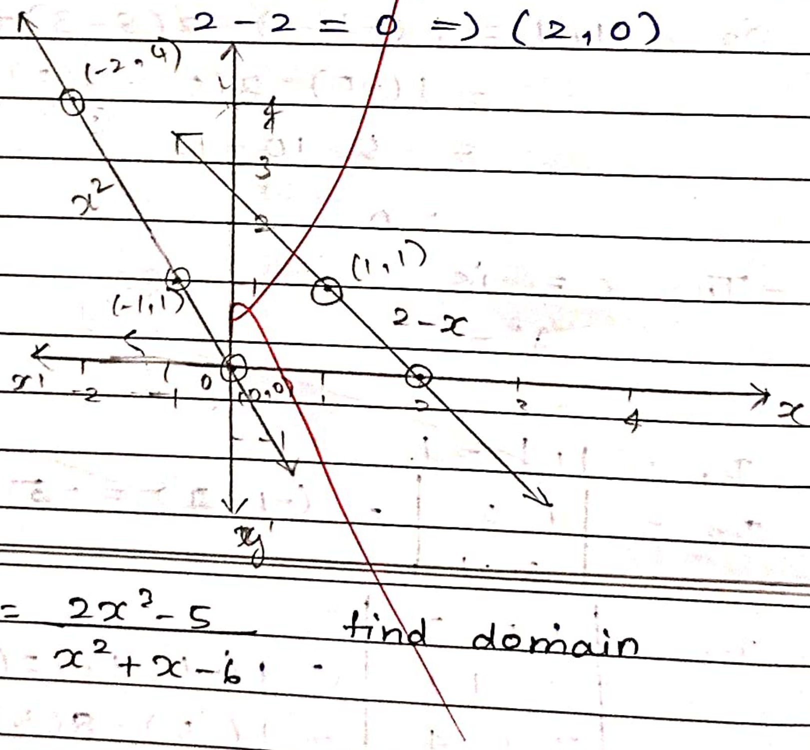
$x = 0 \Rightarrow (0, 0)$

$2 - x \Rightarrow x = 1$

$2 - 1 = 1 \Rightarrow (1, 1)$

$x = 2$

$2 - 2 = 0 \Rightarrow (2, 0)$



6. $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$ find domain

$f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$

$= \dfrac{2x^3 - 5}{(x+3)(x-2)}$

$x = -3, \quad x = 2$

-6

3 -2

+1

The domain of the function is,

$(-\infty, -3) \cup (3, 2) \cup (2, \infty)$

7. Diff $y = x \tan(\sqrt{x})$

$$y = x \tan(\sqrt{x})$$

$$\frac{dy}{dx} = x \cdot \frac{1}{1+(\sqrt{x})^2} \cdot \frac{d}{dx}(\sqrt{x}) + \tan\sqrt{x}\,(1)$$

$$= \frac{x}{1+x} \cdot \frac{1}{2\sqrt{x}} + \tan\sqrt{x}$$

$$\frac{dy}{dx} = \frac{\sqrt{x} \times \cancel{\sqrt{x}}}{1+x} \times \frac{1}{2\cancel{\sqrt{x}}} + \tan\sqrt{x}$$

$$= \frac{\sqrt{x}}{2(1+x)} + \tan\sqrt{x}.$$

8. $u = x - y$, $v = y - z$, $w = z - x$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$

Soln

$u = x - y$	$v = y - z$	$w = z - x$
$\frac{\partial u}{\partial x} = 1$	$\frac{\partial v}{\partial x} = 0$	$\frac{\partial w}{\partial x} = -1$
$\frac{\partial u}{\partial y} = -1$	$\frac{\partial v}{\partial y} = 1$	$\frac{\partial w}{\partial y} = 0$
$\frac{\partial u}{\partial z} = 0$	$\frac{\partial v}{\partial z} = -1$	$\frac{\partial w}{\partial z} = 1$

By Jacobian's method

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= 1(1-0)(-$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} & \frac{\partial w}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} & \frac{\partial w}{\partial z} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 1(1-0) - 0 - 1(1-0)$$

$$= 1(1) - 1(1)$$

$$= 0$$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = 0$$

9. $z = x^2 + y^2 \quad x = t^2 \quad y = 2at \quad \frac{dz}{dt}$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

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$$= 2x \cdot 2t + 2y \cdot 2a$$
$$= 2(t^2)2t + 2(2at) \cdot 2a$$
$$\frac{dz}{dt} = 4t^3 + 8a^2t$$

10. $u = x^2 + y^2 + 2xy$ Verify Euler's theorem.

$$u = x^2 + y^2 + 2xy$$

$$u = x^2\left(1 + \frac{y^2}{x^2} + \frac{2y}{x}\right)$$

If u is a homogenous function of degree 2

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = nu$$

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 2u$$

$$\frac{\partial u}{\partial x} = 2x + 2y \Rightarrow x \cdot \frac{\partial u}{\partial x} = x(2x + 2y)$$

$$\frac{\partial u}{\partial y} = 2y + 2x \Rightarrow y \cdot \frac{\partial u}{\partial y} = y(2y + 2x)$$

$$x(2x + 2y) + y(2y + 2x) = 2u$$

$$= 2(x^2 + xy) + 2(y^2 + xy)$$
$$= 2[x^2 + xy + y^2 + xy]$$
$$= 2[x^2 + 2xy + y^2]$$

$$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 2u$$

Hence Euler's theorem is Verified.

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PART – B

11. (a) $3x^2+5y^2+3z^2-2yz+2zx-2xy$

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

The characteristic eqn is $|A-\lambda I|=0$

(ie) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 3+5+3 = 11$

$$S_2 = \begin{vmatrix} 5 & -1 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ -1 & 5 \end{vmatrix}$$

$= (15-1)+(9-1)+(15-1)$

$= 14+8+14$

$S_2 = 36$

$S_3 = |A| = 3(15-1)+1(-3+1)+1(1-5)$

$= 3(14)-2-4$

$= 42-2-4$

$S_3 = 36$

2	1	-11	36	-36
	0	2	-18	36
	1	-9	18	0

The C.E is

$\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$

$\lambda = 2,\ \lambda^2 - 9\lambda + 18 = 0$

$\lambda = 2,\ \lambda = 3,\ \lambda = 6$

18

$-6 \quad -3$

-9

The eigenvalues are $\lambda = 2, 3, 6$

To find eigenvectors,

case (i) If $\lambda = 2$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} 3-2 & -1 & 1 \\ -1 & 5-2 & -1 \\ 1 & -1 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
$$-x_1 + 3x_2 - x_3 = 0$$
$$x_1 - x_2 + x_3 = 0$$

$$\begin{array}{cccccc} & x_1 & & x_2 & & x_3 \\ 3 & & -1 & & -1 & & 3 \\ -1 & & 1 & & 1 & & -1 \end{array}$$

$$\frac{x_1}{3-1} = \frac{x_2}{-1+1} = \frac{x_3}{1-3}$$

$$\frac{x_1}{2} = \frac{x_2}{0} = \frac{x_3}{-2}$$

$$\frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{-1}$$

The eigenvectors of $\lambda = 2$ is $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

Case (ii) If $\lambda = 3$

$(A - \lambda I)x = 0$

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$$\begin{bmatrix} 3-3 & -1 & 1 \\ -1 & 5-3 & -1 \\ 1 & -1 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$0x_1 - x_2 + x_3 = 0$$
$$-x_1 + 2x_2 - x_3 = 0$$
$$x_1 - x_2 + 0x_3 = 0$$

$$\begin{array}{ccccccc} & & x_1 & & x_2 & & x_3 \\ -1 & & 0 & & 1 & & -1 \\ 2 & & -1 & & -1 & & 2 \end{array}$$

$$\frac{x_1}{1-0} = \frac{x_2}{0+1} = \frac{x_3}{2-1}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$$

The eigenvectors of $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Case (iii) If $\lambda = 6$

$$(A - \lambda I)X = 0$$

$$\begin{bmatrix} 3-6 & -1 & 1 \\ -1 & 5-6 & -1 \\ 1 & -1 & 3-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3x_1 - x_2 + x_3 = 0$$
$$-x_1 - x_2 - x_3 = 0$$
$$x_1 - x_2 - 3x_3 = 0$$

$$\begin{array}{cccccccc} & x_1 & & x_2 & & x_3 & \\ -1 & & -3 & & 1 & & -1 \\ -1 & & 1 & & -3 & & +1 \end{array}$$

$$\frac{x_1}{-1-3} = \frac{x_2}{9-1} = \frac{x_3}{-1-3}$$

$$\frac{x_1}{-4} = \frac{x_2}{8} = \frac{x_3}{-4}$$

$$\frac{x_1}{-1} = \frac{x_2}{2} = \frac{x_3}{-1}$$

The eigenvectors of $\lambda = 6$ is $\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ or $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

The eigenvector matrix

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{\sqrt{1^2+1^2+1^2}} & \frac{1}{\sqrt{1^2+1^2+1^2}} & \frac{1}{\sqrt{1^2+1^2+1^2}} \\ 0 & \frac{1}{\sqrt{1^2+(-2)^2}} & \frac{-2}{\sqrt{1^2+(-2)^2}} \\ \frac{-1}{\sqrt{(-1)^2+1^2+1^2}} & \frac{1}{\sqrt{(1)^2+1^2+1^2}} & \frac{1}{\sqrt{(-1)^2+1^2+1^2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

Diagonaliced matrix

$$D = N^T A N$$

$$D = \begin{bmatrix} \frac{1}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Cano·nical form

$$C = Y^T D Y$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \qquad Y^T = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}$$

$$C = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= \begin{bmatrix} 2y_1 & 3y_2 & 6y_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

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$C = [2y_1^2 + 3y_2^2 + 6y_3^2]$

Nature of the matrix,

All eigenvalues are positive

$\therefore$ The nature of the matrix is positive definite

Rank: (r) No. of non-zero rows,

$\boxed{r = 3}$

Index : No. of positive element

$\boxed{P = 3}$

Signature : $s = 2P - r$

$= 2(3) - 3$

$= 6 - 3$

$\boxed{s = 3}$

12. (b) $Q = 2x_1x_2 + 2x_2x_3 + 2x_1x_3$

$$Q = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The characteristic eqn is $|A - \lambda I| = 0$

(ie) $\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$

$s_1 = 0$

$s_2 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

$= (0-1) + (0-1) + (0-1)$

$= -1 - 1 - 1 = -3$

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$S_3 = |A| = 0 - 1(0-1) + 1(1-0)$

$= 0 - 1(-1) + 1$

$= 1 + 1 = 2$

The C.E is

$\lambda^3 - 0\lambda^2 + (-3)\lambda - 2 = 0$

$\lambda^3 - 0\lambda^2 - 3\lambda - 2 = 0$

$\lambda = -1,\ \lambda^2 - \lambda - 2 = 0$

-1	1	0	-3	-2
	0	-1	1	2
	1	-1	-2	0

$\lambda = -1,\ (\lambda+1)(\lambda-2) = 0$

$\lambda = -1,\ \lambda = -1,\ \lambda = 2$

To find eigen vector

Case(i) If $\lambda = 2$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 0-2 & 1 & 1 \\ 1 & 0-2 & 1 \\ 1 & 1 & 0-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + x_3 = 0$

$x_1 - 2x_2 + x_3 = 0$

$x_1 + x_2 - 2x_3 = 0$

x_1	x_2	x_3	
1	-2	1	
1	1	-2	1

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$$\frac{x_1}{1+2} = \frac{x_2}{4-1} = \frac{x_3}{1+2}$$

$$\frac{x_1}{3} = \frac{x_2}{3} = \frac{x_3}{3}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$$

The eigenvector of $\lambda = 2$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Case (ii) If $\lambda = -1$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 0+1 & 1 & 1 \\ 1 & 0+1 & 1 \\ 1 & 1 & 0+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$

The three equations are same

So we assume $x_1 = 0 \quad x_2 = 1$

$$0 + 1 + x_3 = 0$$
$$x_3 = -1$$

The eigenvector of $\lambda = -1$ is $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

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Case (iii) If $\lambda = -1$

$(A - \lambda I)X = 0$

$$\begin{bmatrix} 0+1 & 1 & 1 \\ 1 & 0+1 & 1 \\ 0 & 0 & 1+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$
$$x_1 + x_2 + x_3 = 0$$

Three equations are same

$\therefore$ We assume $x_1 = 1$ $\quad x_3 = 0$

$$1 + x_2 + 0 = 0$$
$$x_2 = -1$$

The eigenvector of $\lambda = -1$ is $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

The eigenvector matrix is

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Normalised matrix

$$N = \begin{bmatrix} \frac{1}{\sqrt{1^2+1^2}} & 0 & \frac{1}{\sqrt{1^2+1^2}} \\ \frac{1}{\sqrt{1^2+1^2+(-1)^2}} & \frac{1}{\sqrt{1^2+1^2+(-1)^2}} & \frac{-1}{\sqrt{1^2+1^2+1^2}} \\ \frac{1}{\sqrt{1^2+(-1)^2}} & \frac{-1}{\sqrt{1^2+(-1)^2}} & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$N^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & 0 \end{bmatrix}$$

Diagonised matrix

$$D = N^T A N$$

$$D = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Canonical form,

$$c = Y^T D Y$$

$$c = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$c = \begin{bmatrix} 2y_1 & -y_2 & -y_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$c = \begin{bmatrix} 2y_1^2 - y_2^2 - y_3^2 \end{bmatrix}$$

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13.

(b) i) $f(x) = \sqrt{x} - \sqrt[4]{x}$

$$f(x) = (x)^{1/2} - (x)^{1/4}$$

$$f'(x) = \frac{1}{2}(x)^{1/2 - 1} - \frac{1}{4}(x)^{1/4 - 1}$$

$$= \frac{1}{2}(x)^{-1/2} - \frac{1}{4}(x)^{-3/4}$$

$$= \frac{1}{2} x^{-3/4} \left[x^{1/4} - \frac{1}{2} \cancel{x} \right]$$

$$= \frac{1}{2} x^{-3/4} \left[\frac{2x^{1/4} - 1}{2} \right]$$

$$= \frac{1}{4} x^{-3/4} \left[2x^{1/4} - 1 \right]$$

To find critical point

$$f'(x) = 0$$

$$\frac{1}{2} x^{-1/2} - \frac{1}{4} x^{-3/4} = 0$$

$$\frac{1}{4} x^{-3/4} \left[2x^{1/4} - 1 \right] = 0$$

$$2x^{1/4} - 1 = 0$$

$$2x^{1/4} = 1$$

$$x^{1/4} = \frac{1}{2}$$

$$x = \left(\frac{1}{2^4} \right) = \frac{1}{16}$$

$$x = \frac{1}{16}$$

$$f\left(\tfrac{1}{16}\right) = \sqrt{\tfrac{1}{16}} - \sqrt[4]{1/16}$$

$$= \frac{1}{4} - \frac{1 \times 2}{2 \times 2} = \frac{1 - 2}{4} = \frac{-1}{4}$$

$f(\frac{1}{16})$ is maximum.

$$f(\tfrac{1}{16}) = \frac{-1}{4} < 0$$

$$f''(x) = \frac{1}{2}\left(\frac{-1}{2}\right) x^{-1/2-1} - \frac{1}{4}\left(\frac{-3}{4}\right) x^{-3/4-1}$$

$$= \frac{1}{2} \times \frac{-1}{2} x^{-\frac{3}{2}} - \frac{1}{4} \times \frac{-3}{4} x^{-7/4}$$

$$= \frac{-1}{4} x^{-3/2} + \frac{3}{16} x^{-7/4}$$

$$= \frac{1}{4} x^{-7/4} \left[-x^{1/4} + \frac{3}{4} \right]$$

$$f''(16) = \frac{1}{4}\left(\frac{1}{16}\right)^{-7/4} \left[-\left(\frac{1}{16}\right)^{1/4} + \frac{3}{4} \right]$$

$$= \frac{1}{4} \times \left(\frac{1}{2^{-7}}\right) \left[-\frac{1 \times 2}{2 \times 2} + \frac{3}{4} \right]$$

$$= \frac{1}{\cancel{4}} \times \overset{\overset{8}{\cancel{32}}}{\cancel{128}} \times \left(\frac{+1}{\cancel{4}}\right)$$

$$= 8 > 0$$

$\therefore$ f is maximum.

(ii) $$y = \frac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$$

Take log on both sides

$$\log y = \frac{\log\left(\dfrac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}\right)}{\cancel{\log}}$$

$$\log \frac{a}{b} = \log a - \log b$$

$$\log ab = \log a + \log b$$

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$$\log y = \log x^{3/2} + \log \sqrt{x^2+1} - \log [(3x+2)^5]$$

~~$\log y = 3$~~ Diff with respect to x

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2} \cdot \frac{1}{x} + \frac{1}{\sqrt{x^2+1}}$$

$$\log y \Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{3}{2}\log x + \log \sqrt{x^2+1} \cdot 2 - 5\log(3x+2)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2} \times \frac{1}{x} + \frac{1}{2\sqrt{x^2+1}} \times 2x - 5\frac{1}{3x+2}\times 3$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2x} + \frac{2x}{2\sqrt{x^2+1}} - \frac{15}{3x+2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3}{2x} + \frac{x}{\sqrt{x^2+1}} - \frac{15}{3x+2}$$

$$\frac{dy}{dx} = y\left[\frac{3}{2x} + \frac{x}{\sqrt{x^2+1}} - \frac{15}{3x+2}\right]$$

$$\frac{dy}{dx} = \frac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}\left[\frac{3}{2x} + \frac{x}{\sqrt{x^2+1}} - \frac{15}{3x+2}\right]$$

14

(b) (i) $u = \log(x^2+y^2+z^2)$ find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$

Sol

$$u = \log(x^2+y^2+z^2)$$

$$\frac{\partial u}{\partial x} = \frac{1}{(x^2+y^2+z^2)}(2x)$$

$$\frac{\partial u}{\partial x} = \frac{2x}{(x^2+y^2+z^2)}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^2+y^2+z^2)(2) - (2x)(2x)}{(x^2+y^2+z^2)^2}$$

$$= \frac{2x^2+2y^2+2z^2-4x^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{-2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial u}{\partial y} = \frac{1}{x^2+y^2+z^2}(2y)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{(x^2+y^2+z^2)(2) - (2y)(2y)}{(x^2+y^2+z^2)^2}$$

$$= \frac{2x^2+2y^2+2z^2-4y^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{-2y^2+2x^2+2z^2}{(x^2+y^2+z^2)^2}$$

$$\frac{\partial u}{\partial z} = \frac{1}{x^2+y^2+z^2}(2z)$$

$$\frac{\partial^2 u}{\partial z^2} = \frac{(x^2+y^2+z^2)(2) - (2z)(2z)}{(x^2+y^2+z^2)^2}$$

$$= \frac{2x^2+2y^2+2z^2-4z^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{-2z^2+2x^2+2y^2}{(x^2+y^2+z^2)^2}$$

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$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-2x^2+2y^2+2z^2}{(x^2+y^2+z^2)^2} + \frac{-2y^2+2x^2+2z^2}{(x^2+y^2+z^2)^2} + \frac{-2z^2+2x^2+2y^2}{(x^2+y^2+z^2)^2}$$

$$= \frac{-\cancel{2x^2}+2y^2+\cancel{2z^2}-\cancel{2y^2}+2z^2+2x^2+\cancel{2x^2}-\cancel{2z^2}+\cancel{2y^2}}{(x^2+y^2+z^2)^2}$$

$$= \frac{2(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{\cancel{2}}}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{(x^2+y^2+z^2)}$$

(ii) find maxima and minima

$$f(x, y) = x^3y^2(1-x-y)$$

Soln

$$f(x, y) = x^3y^2(1-x-y)$$

$$= x^3y^2 - x^4y^2 - x^3y^3$$

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 4x^3y^2 - 3x^2y^3$$

$$\frac{\partial f}{\partial y} = 2x^3y - 2x^4y - 3x^3y^2$$

$$\frac{\partial^2 f}{\partial x^2} = 6xy^2 - 12x^2y^2 - 6xy^3$$

$$\frac{\partial^2 f}{\partial y^2} = 2x^3 - 2x^4 - 6x^3y$$

$$\frac{\partial^2 f}{\partial x \partial y} = 6x^2y - 8x^3y - 9x^2y^2$$

From ∂f $\quad 3x^2y^2 - 4x^3y^2 - 3x^2y^3$

$$= x^2y^2(3 - 4x - 3y)$$

$$= x^2y^2(3 - 4x - 3y)$$

$x^2y^2 = 0 \qquad 3 - 4x - 3y = 0$

$x = 0$ and $y = 0 \qquad 4x + 3y = 3$ — (1)

From $\frac{\partial f}{\partial y} \Rightarrow 2x^3y - 2x^4y - 3x^3y^2$

$$= x^3y(2 - 2x - 3y)$$

$x^3y = 0 \qquad 2 - 2x - 3y = 0$

$x = 0$ and $y = 0 \qquad 2x + 3y = 2$ — (2)

Solving eqn (1) & (2)

$$4x + 3y = 3$$
$$2x + 3y = 2$$
$$(-) \quad (-) \quad (-)$$

$$2x = 1$$

$$\boxed{x = \frac{1}{2}}$$

Sub $x = \frac{1}{2}$ in (1)

$$4\left(\frac{1}{2}\right) + 3y = 3$$
$$2 + 3y = 3$$
$$3y = 3 - 2$$
$$3y = 1$$
$$\boxed{y = \frac{1}{3}}$$

The stationary points are $(0,0)$ and $\left(\frac{1}{2}, \ldots\right.$

$AC - B^2 = 0$

$A = 6\times\left(\frac{1}{2}\right)\left(\frac{1}{9}\right) - 12\left(\frac{1}{2}\right)^2\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{2}\right)\left(\frac{1}{3}\right.$

$= \frac{1}{3} - 12\times\frac{1}{4}\times\frac{1}{9} - 6\times\frac{1}{2}\times\frac{1}{27}$

$= \frac{1}{3} - \frac{1}{3} - \frac{1}{9} \Rightarrow A = \frac{-1}{9} < 0$

$C = 2\times\frac{1}{8} - 2\times\left(\frac{1}{2}\right)^4 - 6\left(\frac{1}{2}\right)^3\left(\frac{1}{3}\right)$

$= \frac{1}{4} - 2\times\frac{1}{16} - 6\times\frac{1}{8}\times\frac{1}{3}$

$= \frac{1}{4} - \frac{1}{8} - \frac{1}{4} \Rightarrow C = \frac{-1}{8} < 0$

$B^2 = 36x^4y^2 - 64x^5y^2 - 81x^4y^4$

$= 36\times\left(\frac{1}{2}\right)^4\left(\frac{1}{3}\right)^2 - 64\left(\frac{1}{2}\right)^5\left(\frac{1}{3}\right)^2 - 81\left(\frac{1}{2}\right)^4 \ldots$

$= 36\times\frac{1}{16}\times\frac{1}{9} - 64\times\frac{1}{32}\times\frac{1}{9} - 81\times\frac{1}{16}\times \ldots$

$= \frac{1}{4} - \frac{2}{9} - \frac{1}{16}$

$= \frac{1\times 36}{4\times 36} - \frac{2\times 16}{9\times 16} - \frac{1\times 9}{16\times 9}$

$= \frac{36 - 32 - 9}{144}$

$= \frac{-5}{144} < 0$

$AC - B^2 = 0$

$$= \frac{-1}{9} \times \frac{-1}{8} + \frac{5}{144}$$

$$= \frac{1}{72} + \frac{5}{144}$$

$$= \frac{2+5}{144} = \frac{7}{144} > 0$$

$$f\left(\frac{1}{2}, \frac{1}{3}\right) = \left(\frac{1}{2}\right)^3 \left(\frac{1}{3}\right)^2 \left[1 - \frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{1}{8} \times \frac{1}{9} \left[1 - \frac{1}{2} - \frac{1}{3}\right]$$

$$= \frac{1}{72} \left[\frac{6-3-2}{6}\right]$$

$$= \frac{1}{72} \left[\frac{1}{6}\right]$$

$$= \frac{1}{432} > 0$$

$\therefore f\left(\frac{1}{2}, \frac{1}{3}\right)$ is local maximum.

15.
(b) i) $u = \sin^{-1}\left(\frac{x^3 - y^3}{x+y}\right)$ P.T $x \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} = 2 \tan u$

$$u = \sin^{-1}\left(\frac{x^3 - y^3}{x+y}\right)$$

$$\sin u = \left(\frac{x^3 - y^3}{x+y}\right)$$

$$\sin u = z$$

$$z = \left(\frac{x^3 - y^3}{x+y}\right)$$

$$= \frac{x^3\left(1-\frac{y^3}{x^3}\right)}{x\left(1+\frac{y}{x}\right)}$$

$$= \frac{x^2\left(1-\frac{y^3}{x^3}\right)}{\left(1+\frac{y}{x}\right)}$$

By Euler's theorem,

z is an homogenous function of degree 2.

$$x.\frac{\partial z}{\partial x} + y.\frac{\partial z}{\partial y} = nz$$

$$x.\frac{\partial z}{\partial x} + y.\frac{\partial z}{\partial y} = 2z$$

$$x.\cos u.\frac{\partial u}{\partial x} + y.\cos u.\frac{\partial u}{\partial y} = 2\sin u$$

$\div \cos u$

$$x.\frac{\cos u}{\cos u}.\frac{\partial u}{\partial x} + y.\frac{\cos u}{\cos u}\frac{\partial u}{\partial y} = 2\frac{\sin u}{\cos u}$$

$$x.\frac{\partial u}{\partial x} + y.\frac{\partial u}{\partial y} = 2\tan u$$

Hence proved.

(ii) Using Taylor's series

$$F(x, y) = x^2y + \sin y + e^x \qquad (1, \pi)$$

$$F(x, y) = x^2y + \sin y + e^x$$

$$(a, b) = (1, \pi)$$

$$F(1,\pi) = 1^2\pi + \sin\pi + e^1$$
$$= \pi + 0 + e$$
$$= \pi + e$$

$$f_x(x,y) = 2xy + e^x \qquad f_x(1,\pi) = 2(1)(\pi) + e^1$$
$$= 2\pi + e$$

$$f_y(x,y) = x^2 + \cos y \qquad f_y(1,\pi) = 1 + \cos\pi$$
$$= -1$$

$$f_{xx}(x,y) = 2y + e^x \qquad f_{xx}(1,\pi) = 2\pi + e^1$$
$$= 2\pi + e$$

$$f_{yy}(x,y) = -\sin y \qquad f_{yy}(1,\pi) = -\sin(\pi)$$
$$= 0$$

$$f_{xxy}(x,y) = 2 + e^x \qquad f_{xxy}(1,\pi) = 2 + e$$

$$f_{yyx}(x,y) = -\cos y \qquad \cancel{f_{yyx}(1,\pi)} = -\cos\pi$$
$$= -1(-1) = 1$$

By using Taylor's series.

$$F(x,y) = (a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right]$$
$$+ \frac{1}{2!}\Big[(x-a)^2 f_{xx}(a,b) + 2(x-a)(y-b)f_{xy}(a,b) + (y-b)^2 f_{yy}(a,b)\Big] + \dots$$

$$= (1,\pi) + \frac{1}{1!}\left[(x-1)(2\pi+e) + (y-\pi)(-1)\right]$$
$$+ \frac{1}{2!}\Big[(x-1)^2(2\pi+e) + 2(x-1)(y-\pi)(2) + (y-\pi)^2(0)\Big] + \dots$$

$$= (1,\pi) + \frac{1}{1!}\Big[(x-1)(2\pi+e) + 2(x-1)(y-\pi)$$
$$(2) + 0\Big] + \ldots$$

PART- A

3. 2×2 matrix $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = -|A|$$
$$8 + 4 + \lambda_3 = (-1-1)$$
$$12 + \lambda_3 = -2$$
$$\lambda_3 = -2 - 12$$
$$\lambda_3 = -14$$

DATE / /

Name : S. Sanmathi.
Department : B.E. CSE.
Class : B.
Slip test : III.

Subject : Matrices and calculus.
Subject code : MA3151.
Date : 26.10.2024.

13/20

S. Sanmathi.

2 marks :

1. $y = (x^3 - 1)^{100}$.

Solution :

Given $y = (x^3 - 1)^{100}$

here $g(x) = 1 - x^3$.

$g'(x) = 0 - 3x^2$.

$= -3x^2$

$f(x) = [g(x)]^{100}$.

$f'(x) = 100\,[g(x)]^{99}\, g'(x)$

$= 100\,(1 - x^3)^{99}\,(-3x^2)$.

2. $f(x) = \dfrac{x-4}{x^2-9}$.

Solution :

Given $f(x) = \dfrac{x-4}{x^2-9}$.

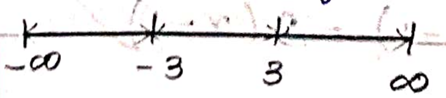
If $x^2 - 9 = 0$.

$x^2 = 9$.

$x = \pm\sqrt{9}$.

$x = \pm 3$.

If $x = \pm 3$ is not defined.



Range?

DATE / /

Domain : $(-\infty,-3) \cup (-3,3) \cup (3,\infty)$.

1. $y = x^2$ at $(1,1)$.

Solution :

Given $y = x^2$

$f(x) = x^2 = (1,1)$.

$$M = f'(a) = \lim_{x \to 1} \frac{f(x) - f(a)}{x-a}$$

$$M = f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1}$$

$$= \lim_{x \to 1} \frac{x^2 - (1)^2}{x-1}$$

$$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

$$= \lim_{x \to 1} \frac{(x+1)}{\cancel{x-1}}$$

$= 1+1$

$\boxed{M = 2.}$

The equation are.

$(y-y_1) = M(x-x_1)$

$(y-1) = 2(x-1)$

DATE / /

$y - 1 = 2x - 2.$

$2x - 2 - y + 1 = 0.$

The tangent equation are.

$\boxed{2x - y - 1 = 0}$

2. $f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2. \end{cases}$

<u>solution:</u>

Given $f(x) = \begin{cases} cx^2 + 2x, & x < 2 \\ x^3 - cx, & x \geq 2. \end{cases}$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} cx^2 + 2x.$

$= c(2)^2 + 2(2).$

$= 4c + 4 \longrightarrow (1).$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^3 - cx$

$= (2)^3 - c(2).$

$= 8 - 2c \longrightarrow (2).$

f is continuous at $(-\infty, \infty)$.
Limit is exist.
left side limit and right side limit is equal.

L.S.L = R.S.L.

$4c + 4 = 8 - 2c$

$4c + 2c = 8 - 4$

DATE / /

$$6c = 4.$$

$$c = \frac{4}{6}$$

$$\boxed{c = \frac{2}{3}}$$

3. $\lim_{x \to 0} \frac{\sin x}{x}$.

Solution:

Given $\lim_{x \to 0} \frac{\sin x}{x}$

$$f(x) = \frac{\sin x}{x}$$

$$f(x) = \lim_{x \to 0} \frac{\sin x}{x}$$

$$= \lim_{x \to 0} \frac{\sin 0}{0}$$

$$= \left(\frac{0}{0}\right).$$

$$= \lim_{x \to 0} \frac{\cos x}{x}.$$

$$= \lim_{x \to 0} \frac{\cos 0}{1}$$

$$= \frac{1}{1}$$

$$= 1.$$

DATE / /

4. $\lim_{x \to \frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$.

Solution:

Given, $\lim_{x \to \frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$.

$= \lim_{x \to \frac{\pi}{2}} \frac{1+\cos\pi}{(\pi-\pi)^2}$.

$= \lim_{x \to \frac{\pi}{2}} \frac{1-1}{0}$.

$= \frac{1-1}{0} = \frac{0}{0}$.

$\lim_{x \to \frac{\pi}{2}} = \frac{-2\sin 2x}{(\pi-2x)^2}$

$= \frac{-2\sin\pi}{(\pi-\pi)^2}$.

$= \frac{-2}{4}$.

$= \frac{-1}{2}$.

Name : M. Thirumurugan
Department : CSE
Register no : T3024~~24~~104107
Slip Test : 4

Class : Sec 'B'
Subject : Matrices and Calculus
Sub code : MA3151
Date : 20/11/24

1) State mean value Theorem

Mean Value Theorem, In other words MVT is a point of c in interval (a, b) The instaneous The rate of Change of function $f'(c)$ is equal to rate of change of average function The entire of [a, b]

2) Critical point $y = 5x^3 - 6x$

$f(x) = 5x^3 - 6x$

$f'(x) = 15x^2 - 6$

$f'(x) = 0$

$\cancel{f'(x)} = 15x^2 - 6 = 0$

$15x^2 = 6$

$x^2 = \frac{\cancel{6}\,2}{\cancel{15}\,5}$

$x^2 = \sqrt{\cancel{2/5}}\ \frac{2}{5}$

$x = \pm\sqrt{\frac{2}{5}}$

8 mark

1) Given

$x^2 + xy + y^2 = 1$

$x = 1$

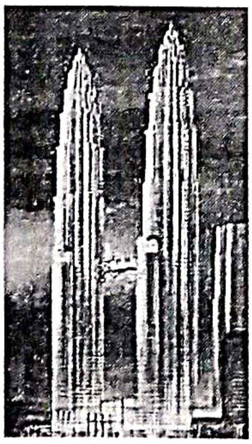
$\frac{dy}{dx} = 0(x^2 + xy + x^3) = 0$

$\frac{d}{dx} = (x - y)$

Content beyond the Syllabus

- The eigenvalues are used to determine the natural frequencies (or eigenfrequencies) of vibration, and the eigenvectors determine the shapes of these vibrational modes.Most structures from buildings to bridges have a natural frequency of vibration. Eigenvalues can also be used to test for cracks or deformities in structural components used for construction. Model population growth using an age transition matrix and an age distribution vector, and find a stable age distribution vector.Use a matrix equation to solve a system of first-order linear differential equations.Find the matrix of a quadratic form and use the Principal AxesTheorem to perform a rotation of axes for a conic and a quadric surface.
- There are many applications of sequences. To solve problems involving sequences, it is a good strategy to list the first few terms, and look for a pattern that aids in obtaining the general term. When the general term is found, then one can find any term in the sequence without writing all the preceding terms. Sequences are useful in our daily lives as well as in higher mathematics. For example, the interest portion of monthly payments made to pay off an automobile or home loan, and the list of maximum daily temperatures in one area for a month is sequences.
- There was not a good enough understanding of how the Earth, stars and planets moved with respect to each other. Calculus (differentiation and integration) was developed to improve this understanding. We use the derivative to determine the maximum and minimum values of particular functions (e.g. cost, strength, amount of material used in a building, profit, loss, etc.).

1. Derivatives are met in many engineering and science problems, especially when modeling the behavior of moving objects.
2. It is used ECONOMIC a lot, calculus is also a base of economics. In economics, calculus is used to compute marginal cost and marginal revenue, enabling economists to predict maximum profit in a specific setting.
3. The **Petronas Towers** in Kuala Lumpur experience high forces due to winds. **Integration** was used to design the building for strength.



4. The **Sydney Opera House** is a very unusual design based on slices out of a ball. Many **differential equations** (one type of integration) were solved in the design of this building.

5. Historically, one of the first uses of integration was in finding the **volumes of wine-casks** (which have a curved surface).

6. It is used in history, for predicting the life of a stone.
7. The newbie, **PID controller** is a control loop feedback mechanism (controller) widely used in industrial control systems.
8. Applications of the Indefinite Integral shows how to find displacement (from velocity) and velocity (from acceleration) using the indefinite integral.

- Taylor's series is an essential theoretical tool in computational science and approximation. One application is to use series to approximate solutions to differential equations. In many cases, solving for a given variable outright can be very difficult or even impossible. Representing the variable as a Taylor Series, it is far easier to approximate a solution around a particular point.

- One of the major applications of multiple integrals in engineering, particularly structures and mechanics, is the determination of properties of plane (i.e. effectively 2-D) and solid (i.e. 3-D) bodies – their volume, mass, centre of gravity, moment of inertia, etc.

1. In mechanics, the moment of Inertia is calculated aş the volume integral (triple integral) of the density weighed with the square of the distance from the axis.
2. In electromagnetism, Maxwell's equations can be written using multiple integrals to calculate the total magnetic and electric fields.

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DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Question Paper

Tutorial – 01			Date of Issue:	09.10.2024	Marks	10
Course code	MA3151	Course Title	Matrices And Calculus			
Year	I	Semester/Section	I / All Branches	Date of Submission:	23.10.2024	

Q.No	Questions	CO
1	Find the domain of the function $f(x) = \frac{2x^3-5}{x^2+x-6}$	CO 2
2	Find the local maxima and minima of $f(x) = \sqrt{x} - \sqrt[4]{x}$	CO 2

Name and Signature of the Faculty Incharge

HoD/S&H

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DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Answer Sheet

Name of the Student : Monisha. N.P

AU Register Number: 732424104067

Tutorial – 01			Date of Issue:	09.10.2024	Marks	10
Course code	MA3151	Course Title	Matrices And Calculus			
Year	I	Semester/Section	I / All Branches	Date of Submission:	23.10.2024	

Q.No	Questions	CO
1	Find the domain of the function $f(x) = \frac{2x^3-5}{x^2+x-6}$	CO 2
2	Find the local maxima and minima of $f(x) = \sqrt{x} - \sqrt[3]{x}$	CO 2

Mark Allocation

Rubrics	Marks Allocated	Marks obtained
Problem solving approach	6	6
Correctness of Answer	2	2
Timely submission	2	2
Total marks	10	10

G. Saravanakumar.

23/10/24

Name and Signature of the Faculty Incharge

HoD/S&H

Find the domain of the function $f(x) = \dfrac{2x^3-5}{x^2+x-6}$

Solution:

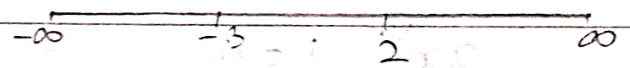
let $x^2+x-6=0$

$(x+3)(x-2)=0$

$x+3=0 \qquad x-2=0$

$x=-3 \qquad x=2$

then the function is not defined



$x \in (-\infty,-3) \cup (-3,2) \cup (2,\infty)$

Find the local maxima and minima of
$f(x) = \sqrt{x} - \sqrt[4]{x}$

Solution:

Given.

$f(x) = \sqrt{x} - \sqrt[4]{x}$

$f(x) = (x)^{1/2} - (x)^{1/4}$

$f'(x) = \frac{1}{2}(x)^{1/2-1} - \frac{1}{4}(x)^{1/4-1}$

$= \frac{1}{2}(x)^{-1/2} - \frac{1}{4}(x)^{-3/4}$

$= \frac{1}{4}x^{-3/4}\left(x^{1/4} - \frac{1}{2}\right)$

$= \frac{1}{2}x^{-3/4}\left(\frac{2x^{1/4}-1}{2}\right)$

$= \frac{1}{4}x^{-3/4}\left(2x^{1/4}-1\right)$

DATE / /

$$= \tfrac{1}{2} x^{-3/4} \left(\frac{2x^{1/4} - 1}{2} \right)$$

$$f'(x) = \tfrac{1}{4} x^{-3/4} (2x^{1/4} - 1)$$

To find critical number

$$f'(x) = 0$$

$$\tfrac{1}{4} x^{-3/4} (2x^{1/4} - 1) = 0$$

$$2x^{1/4} - 1 = 0$$

$$2x^{1/4} = 1$$

$$x^{1/4} = \tfrac{1}{2}$$

$$x = (\tfrac{1}{2})^4$$

$$x = \tfrac{1}{16}$$

The critical number $x = 1/16$

The first derivative test

$$f(1/16) = \sqrt{1/16} - \sqrt[4]{1/16}$$

$$= \left(\frac{1}{4^2}\right)^{1/2} - \left(\frac{1}{2^4}\right)^{1/4}$$

$$= \tfrac{1}{4} - \tfrac{1}{2}$$

$$= \frac{1}{4} - \frac{1}{2} \times \frac{2}{2}$$

$$= \frac{1-2}{4}$$

DATE / /

$= -1/4 < 0$

$f(1/16) = -1/4 < 0$

$\therefore f$ is local minimum

$f(1/16) = -1/4$

The second derivative test

$f''(x) = 1/2(-1/2)\,x^{-1/2-1} - 1/4\left(\frac{-3}{4}\right)x^{-3/4-1}$

$= -1/4\, x^{-7/4}\left(-x^{1/4} \cdot x^{3/4}\right)$

$= 1/4\, x^{-7/4}\left(\frac{-4x^{1/4}+3}{4}\right)$

$= \frac{1}{16} x^{-7/4}\left(-4x^{1/4}+3\right)$

$f''(1/16) = \frac{1}{16}\left(\frac{1}{16}\right)^{-7/4}\left(-4(1/16)^{1/4}+3\right)$

$= 1/16\left(\frac{1}{2^{-7}}\right)\left(-4(1/2)+3\right)$

$= \frac{1}{\cancel{16}_{4}}\left(\cancel{128}^{32\,8}\right)(-2+3)$

$= 8 > 0$

local minimum at $x = 1/16$

The local minimum value of $f(1/16) = 1/4$

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Vijayamangalam, Tiruppur.

DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Question Paper

Tutorial – 02			Date of Issue:	02.12.2024	Marks	10
Course code	MA3151	Course Title	Matrices And Calculus			
Year	I	Semester/Section	I / All Branches	Date of Submission:	13.12.2024	

Q.No	Questions	CO
1	Evaluate $\int_1^2\int_2^5 xy\,dydx$	CO 5
2	Find the volume of sphere $x^2+y^2+z^2=25$ using triple integral	CO 5

Jeevanandham. 13/12/24

Name and Signature of the Faculty Incharge

HoD/S&H

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SASURIE COLLEGE OF ENGINEERING

Vijayamangalam, Tiruppur

DEPARTMENT OF SCIENCE AND HUMANITIES

Tutorial Answer Sheet

Name of the Student : Swetha .N.

AU Register Number: 732424 104104

Tutorial – 02			Date of Issue:	02.12.2024	Marks	10
Course code	MA3151	Course Title	Matrices And Calculus			
Year	I	Semester/Section	I / All Branches	Date of Submission:	13.12.2024	

Q.No	Questions	CO
1	Evaluate $\int_1^2 \int_2^5 xy\, dydx$	CO 5
2	Find the volume of sphere $x^2 + y^2 + z^2 = 25$ *using triple integral*	CO 5

Mark Allocation

Rubrics	Marks Allocated	Marks obtained
Problem solving approach	6	6
Correctness of Answer	2	2
Timely submission	2	2
Total marks	10	10

G. Jeevanantam. 13/12/2024

Name and Signature of the Faculty Incharge

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DATE:

① Evaluate $\int_1^2 \int_2^5 xy \, dy dx$.

Solution

$$\int_1^2 \int_2^5 xy \, dy dx = \int_1^2 x \left[\frac{y^2}{2}\right]_2^5 dx$$

$$= \int_1^2 x \left(\frac{25}{2} - \frac{4}{2}\right) dx$$

$$= \frac{21}{2}\left[\frac{x^2}{2}\right]_1^2$$

$$= \frac{21}{2}\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$= \frac{21}{2}\left[3/2\right]$$

$$= 63/4$$

② Find the volume of sphere (or) find the volume of sphere $x^2 + y^2 + z^2 = 25$ using triple integration (or) find the volume of sphere $x^2 + y^2 + z^2 = 25$

Solution:

the limit are

$x \to 0 \to a$

$y = 0 \to \sqrt{a^2 - x^2}$

$z = 0 \to \sqrt{a^2 + x^2 - y^2}$

SPB

DATE :

Volume of Shere $= V = 8 \iiint dx\,dy\,dz$

$$= 8 \int_0^a \int_0^{\sqrt{a^2+x^2}} \int_0^{\sqrt{a^2+x^2-y^2}} dx\,dy\,dz$$

$$= 8 \int_0^a \int_0^{\sqrt{a^2+x^2}} [z]_0^{\sqrt{a^2+x^2-y^2}} dx\,dy$$

$$= 8 \int_0^a \int_0^{\sqrt{a^2+x^2}} \left(\sqrt{a^2+x^2-y^2}\right) dx\,dy$$

WKT.

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}(x/a) + C$$

$$= 8 \int_0^a \left[y/2 \sqrt{a^2+x^2y^2} + \frac{a^2-x^2}{2} \sin^{-1}\left(\frac{y}{\sqrt{a^2-x^2}}\right) \right]_0^{\sqrt{a^2}}$$

$$= 8 \int_0^a \left\{ \frac{\sqrt{a^2-x^2}}{2} \sqrt{a^2-x^2-(a^2-x^2)} + \frac{a^2-x^2}{2}\left(\frac{\sqrt{a^2-x^2}}{\sqrt{a^2-x^2}}\right) \right] - 0 \Big\}$$

$$= 8 \int_0^a \left\{ \frac{\sqrt{a^2-x^2}}{a^2}(0) + \frac{a^2-x^2}{2} \sin^{-1}(1) \right\} dx$$

$$= \frac{8\pi}{4} \int_0^a \left(a^2-x^2\right) dx$$

DATE :

$$= \frac{8\pi}{4}\left(a^3 - \frac{a^3}{3}\right) - 10)$$

$$= \frac{8\pi}{4}\left(\frac{2a^3}{3}\right)$$

$$= \frac{2\pi}{2}\left(\frac{2a^3}{3}\right)$$

$$V = \frac{4\pi a^3}{3}$$

If $a = 5$

$$V = \frac{4\pi 5^3}{3}$$

$$V = 4\pi 2^3$$

$\Rightarrow V = 52\pi$

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SASURIE
COLLEGE OF ENGINEERING

ASSIGNMENT SCHEDULE

Subject : Matrices and Calculus

Faculty : G. Jeevanan Tam.

Semester : I

Year : I

Department : CSE

S.No	Particulars	Target Date
1	eigen values and eigen vector	2[illegible].10.24
2	Differential and Integral.	02.12.2024.
3	—	—
4	—	—

	Prepared by	Verified by
Sign		
Name	G. Jeevanan Tam.	H. Sathya
	Faculty	HOD

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Vijayamangalam, Tiruppur.

DEPARTMENT OF SCIENCE AND HUMANITIES

Assignment Question Paper

Assignment – 01				Date of Issue:	05.10.2024	Marks	10
Course code	MA3151	Course Title	MATRICES AND CALCULUS				
Year	I	Semester/Section	I / B	Date of Submission:	21.10.24		

Q.No	Questions	CO
1	Find the eigen values & eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$.	C101.1
2	Find the eigen values & eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.	C101.1

G. Jeevanan / Cum.

21/10/2024

Name and Signature of the Faculty Incharge

HoD/S&H

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Vijayamangalam, Tiruppur

DEPARTMENT OF SCIENCE AND HUMANITIES

Assignment Answer Sheet

Name of the Student : M. Vignesh

AU Register Number: 732424101112

Assignment – 01			Date of Issue:	05.10.2024	Marks	10
Course code	MA3151	Course Title	MATRICES AND CALCULUS			
Year	I	Semester/Section	I / B	Date of Submission:	21.10.24	

Q.No	Questions	CO
1	Find the eigen values & eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{pmatrix}$.	C101.1
2	Find the eigen values & eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.	C101.1

Mark Allocation

Rubrics	Marks Allocated	Marks obtained
Content Quality	6	6
Presentation Quality	2	1
Timely submission	2	2
Total marks	10	9

G. Jeevanantham.
21/10/2024

Name and Signature of the Faculty Incharge

M. B

HoD/S&H

Find the eigen value A if $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$

Solution:-

$S_1 = 3+4+5$

$S_1 = 12$

$S_2 = \begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 6 & 5 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 8 & 4 \end{vmatrix}$

$= (20-0)+(15-0)+(12-0)$

$= 20+15+12$

$S_2 = 47$

$S_3 = 3\begin{vmatrix} 4 & 0 \\ 2 & 5 \end{vmatrix} + 0\begin{vmatrix} 8 & 0 \\ 6 & 5 \end{vmatrix} + 0\begin{vmatrix} 8 & 4 \\ 6 & 2 \end{vmatrix}$

$= 3(20-0)+0+0$

$= 60$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda = 3 \cdot \lambda^2 - 9\lambda + 20 = 0$

$\lambda = 3, \lambda = 4, \lambda = 5$ adj $A = \frac{|A|}{\lambda}$

If λ is an eigen values of the Matrix A then $\frac{|A|}{\lambda}$ is an eigen value Adj A

$|A| = 60$

$\frac{|A|}{\lambda} = \frac{60}{3}, \frac{60}{4}, \frac{60}{5}$

Adj A = 20, 15, 12.

If two eigen value of the Matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$ are equal to each, find the

eigen value of A^{-1}

If A $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

$\lambda 1 = 1, \lambda 2 = 1$

to find $\lambda 3 = ?$

Sum of the e.v = T race of A

$\lambda 1 + \lambda 2 + \lambda 3 = 2+3+2$

$1+1+\lambda 3 = 7$

$2+\lambda 3 = 7$

$\lambda 3 = 7-2$

$\lambda 3 = 5$

$\lambda_1 = 1, \lambda_2 = \cancel{1}, \lambda_3 = 3$

$\lambda 1, \lambda 2, \lambda 3$ is eigen values of A $1/\lambda 1$, $1/\lambda 2$, $1/\lambda 3$ is eigen values of A^{-1} the eigen values of A^{-1}

$1/\lambda_1 = 1/1 = 1$

$1/\lambda_2 = 1/1 = 1$

$1/\lambda_3 = 1/5$.

2marks:-

(1) write the Statement of Caylay - Hamilton theorem:-

Solu:

Every square matrix satisfied its own characteristic equation.

If λ is eigen value of A then prove that λ^2 is eigen of A^2 (2)

Solu:

Given λ is eigen value of n

we know that

$$(A-\lambda I)x=0$$
$$Ax-\lambda Ix=0$$
$$Ax-\lambda x=0$$
$$Ax=\lambda x \longrightarrow (1)$$

Premultiple by A

$$A(Ax)=A\lambda x$$
$$A^2x=A\lambda x$$
$$A^2x=\lambda Ax$$

From equation (1)

$$A^2x=\lambda(\lambda x) \qquad A^2x=\lambda^2 x$$

λ^2 is eigen value of A^2

Hence proved.

8marks:-

Find the eigen value and eigen vector of the Matrix

$$A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{bmatrix}$$

Solu:

Given $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{bmatrix}$

the chraclinstic eauation are $|A - \lambda I| = 0$

(ie) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1+2+3$

$\boxed{S_1 = 6}$

$S_2 = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$

$= (6-4)+(3+4)+(2-0)$

$= 2+7+2$

$\boxed{S_2 = 11}$

$S_3 = 1\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ -4 & 3 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ -4 & 4 \end{vmatrix}$

$= 1(6-4) - 1\cdot(0+4) + 1(0-8)$

$= 1(2) - 1\cdot(4) + 8$

$= 10 - 4$

$\boxed{S_3 = 6}$

Case 1 :-

$\lambda = 1$

$$\begin{bmatrix} 1-1 & 1 & 1 \\ 0 & 2-1 & 1 \\ -4 & 4 & 3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ -4 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + x_2 + x_3 = 0$

$0x_1 + x_2 + x_3 = 0$

$-4x_1 + 4x_2 + 2x_3 = 0$

$$\frac{x_1}{2-4} = \frac{x_2}{-4+0} = \frac{x_3}{0+4}$$

$$\frac{x_1}{-2} = \frac{x_2}{-2} = \frac{x_3}{4}$$

$\div 2$

$$\frac{x_1}{-1} = \frac{x_1}{2} = \frac{x_3}{2}$$

the eigen vector of $\lambda =$ is $\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$

Case 2 :-

$\lambda = 2$

$$\begin{bmatrix} 1-2 & 1 & 1 \\ 0 & 2-2 & 1 \\ -4 & 4 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x_1 + x_2 + x_3 = 0$

$0x_1 + 0x_2 + x_3 = 0$

$-4x_1 + 4x_2 + x_3 = 0$

$$\frac{x_1}{1+0} = \frac{x_2}{0+1} = \frac{x_3}{0+0}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{0}$$

the eigen vector of $\lambda = 2$ is $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Case 3 :-

$\lambda = 3$

$$\begin{bmatrix} 1-3 & 1 & 1 \\ 0 & 2-3 & 1 \\ -4 & 4 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ -4 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + x_3 = 0$

$0x_1 - x_2 + x_3 = 0$

$-4x_1 + 4x_2 + 0x_3 = 0$

$$\frac{x_1}{1+1} = \frac{x_2}{0+2} = \frac{x_3}{2+0}$$

$$\frac{x_1}{2} = \frac{x_2}{2} = \frac{x_3}{2} \Rightarrow \frac{x_1}{1} = \frac{x_1}{1} = \frac{x_3}{1}$$

The eigen vector of $\lambda = 3$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(4) Find the eigen value and eigen vector of the Matrix $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Solution:-

Given $B = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

The characteristic equation

$|A - \lambda I| = 0$

(ie) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1+5+1$

$\boxed{S_1 = 7}$

$S_2 = \begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix}$

$= (5-1) + (1-9) + (5-1)$

$= 4 - 8 + 4$

$= 8 - 8$

$= 0$

$S_3 = 1\begin{vmatrix} 5 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 5 \\ 3 & 1 \end{vmatrix}$

$= 1(5-1) - 1(1-3) + 3(1-15)$

$= 1(4) - 1(-2) + 3(-14)$

$= 4 + 2 - 42$

$S_3 = -36$

The characteristic equation

$\lambda^3 - 7\lambda^2 + 0\lambda + 36 = 0$

$$\begin{array}{c|cccc} 6 & 1 & -7 & 0 & 36 \\ & 0 & 6 & -6 & -36 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$\lambda = 6, \lambda^2 - 6 = 0$

$\lambda = 6, (\lambda - 3)(\lambda + 2) = 0$

$\lambda = 6, \lambda = 3, \lambda = 2$

The eigen value are $\lambda = -2, 3, 6$

Case I :-

If $\lambda = -2$

$[A - \lambda I]x = 0$

$[A - (-2)I]x = 0$

$|A + 2I|x = 0$

$$\begin{bmatrix} 1+2 & 1 & 3 \\ 1 & 5+2 & 1 \\ 3 & 1 & 1+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 3 \\ 1 & 7 & 1 \\ 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3x_1 + x_2 + 3x_3 = 0$

$x_1 + 7x_2 + x_3 = 0$

$3x_1 + x_2 + 3x_3 = 0$

$$\begin{array}{cccc} x_1 & x_2 & & x_3 \\ 1 & 3 & 3 & 1 \\ 7 & 1 & 1 & 7 \end{array}$$

$$\frac{x_1}{(-21)} = \frac{x_2}{3-3} = \frac{x_3}{21-1}$$

$$\frac{x_1}{-20} = \frac{x_2}{0} = \frac{x_3}{20}$$

$\div 20$

$$\frac{x_1}{-1} = \frac{x_2}{0} = \frac{x_3}{1}$$

The eigen vector of $\lambda = -2$ is $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

Case 2:-

$\lambda = 3$

$$\begin{bmatrix} 1-3 & 1 & 3 \\ 1 & 5-3 & 1 \\ 3 & 1 & 1-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 + 3x_3 = 0$

$x_1 + 2x_3 + x_3 = 0$

$3x_1 + x_2 - 2x_3 = 0$

x_1	x_2	x_3	
1	3	-2	1
2	1	1	2

$$\frac{x_1}{1-6} = \frac{x_2}{3+2} = \frac{x_3}{-4-1}$$

$$= \frac{x_1}{-5} = \frac{x_2}{5} = \frac{x_3}{-5}$$

$$\frac{x_1}{-1} = \frac{x_2}{1} = \frac{x_3}{-1}$$

The eigen vector of $\lambda = 3$ is $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

Case 3:- $\lambda = 6$

$$\begin{bmatrix} 1-6 & 1 & 3 \\ 1 & 5-6 & 1 \\ 3 & 1 & 1-6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 1 & 3 \\ 1 & -1 & 1 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x_1 + x_2 + 3x_3 = 0$

$x_1 - x_2 + x_3 = 0$

$3x_1 + x_2 - 5x_3 = 0$

1

$$\frac{x_1}{4} = \frac{x_2}{8} = \frac{x_3}{4}$$

$$\frac{x_1}{1} = \frac{x_2}{2} = \frac{x_3}{1}$$

the eigen vector of $\lambda = 6$ is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(ii) verified Calyley - Hamiltan theorem the Matrix

$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and Find A^{-1}

Given $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

the charactirstic equation $|A - \lambda I| = 0$

$S_1 = 1 + 2 + 1$

$S_1 = 4$

$S_2 = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 7 \\ 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix}$

$= (2-6) + (1-7) + (2-12)$

$= -4 - 6 - 10$

$= -20$

$S_3 = 1 \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} 4 & 3 \\ 1 & 1 \end{vmatrix} + 7 \begin{vmatrix} 4 & 2 \\ 1 & 2 \end{vmatrix}$

$= 1(2-6) - 3(4-3) + 7(8-2)$

$= 1(-4) - 3(1) + 7(6)$

$= -4 - 3 + 42$

$S_3 = 35$

the charactristic equation are

$\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$

by Cayhaly Hamilton theorem every square Matrix satisfiod its own charactristic eqation.

$$A^3 - 4A^2 - 20A - 35I = 0$$

$$A^3 = A^2 A \Rightarrow A^2 = AA$$

$$A^2 = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+12+7 & 3+6+14 & 7+9+7 \\ 4+8+3 & 12+4+6 & 28+6+3 \\ 1+8+1 & 3+4+2 & 7+6+1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix}$$

$$A^2 A = \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix} \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20+92+23 & 60+46+46 & 140+69+23 \\ 15+88+37 & 45+44+74 & 105+66+37 \\ 10+36+14 & 30+18+24 & 70+27+14 \end{bmatrix}$$

$$= \begin{bmatrix} 135 & 152 & 232 \\ 140 & 163 & 208 \\ 60 & 76 & 111 \end{bmatrix}$$

$$AA^2 = 4 \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 80 & 92 & 92 \\ 60 & 88 & 148 \\ 40 & 36 & 56 \end{bmatrix}$$

$$20A = 20\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}20 & 60 & 140\\ 80 & 40 & 60\\ 20 & 40 & 20\end{bmatrix}$$

$$35I = 35\begin{bmatrix}35 & 0 & 0\\ 0 & 35 & 0\\ 0 & 0 & 35\end{bmatrix}$$

$$A^3 - AA^2 - 20A - 35I = 0$$

$$= \begin{bmatrix}135 & 152 & 232\\ 140 & 163 & 208\\ 60 & 76 & 111\end{bmatrix} - \begin{bmatrix}80 & 92 & 92\\ 60 & 88 & 148\\ 40 & 36 & 56\end{bmatrix} - \begin{bmatrix}20 & 60 & 140\\ 80 & 40 & 60\\ 20 & 40 & 20\end{bmatrix} - \begin{bmatrix}35 & 0 & 0\\ 0 & 35 & 0\\ 0 & 0 & 35\end{bmatrix}$$

$$= \begin{bmatrix}135-80-20-35 & 152-92-60-0 & 232-92-40-0\\ 140-60-80-0 & 163-88-40-35 & 208-148-60-0\\ 60-40-20-0 & 76-36-40-0 & 111-56-20-35\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

to Find A^{-1}

the characteristic equation

$$\lambda^3 - 4\lambda^2 - 20\lambda - 35 = 0$$

$$A^3 - 4A^2 - 20A - 35I = 0$$

Premultiple by A^{-1}

$$A^{-1}A^3 - 4A^{-1}A^3 - 20AA^{-1} - 35A^{-1}I = 0$$

$$A^2 - 4A - 20I - 35A^{-1} = 0$$

$$35A^{-1} = A^2 - 4A - 20$$

$A^2 - 4A - 20I - 35A^{-1} = 0$

$35A^{-1} = A^2 - 4A - 20I$

$$35A^{-1} = \begin{bmatrix} 20 & 23 & 23 \\ 15 & 22 & 37 \\ 10 & 9 & 14 \end{bmatrix} - \begin{bmatrix} 4 & 12 & 28 \\ 16 & 8 & 12 \\ 4 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$$

$$A^{-1} = \frac{1}{35} \begin{bmatrix} 20-4-20 & 23-12-0 & 23-28-0 \\ 15-16-0 & 22-8-20 & 37-11-0 \\ 10-4-0 & 9-8-0 & 14-4-20 \end{bmatrix}$$

$$= \frac{1}{35} \begin{bmatrix} -4 & 11 & -5 \\ -1 & -6 & 25 \\ 6 & 1 & 10 \end{bmatrix}$$

verified Cayley - Hamilton theorem of the matrix

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and find A^{-1}

Soln

Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

the characteristic equation are $|A - \lambda I| = 0$

(ie) $\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = 1+2+3$

$S_1 = 6$

$S_2 \begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$= (6-3) + (3-2) + (2-1)$

$= 3+1+1$

$S_2 = 5$

$S_3 = 1 \begin{vmatrix} 2 & -3 \\ -1 & 3 \end{vmatrix} - 1 \begin{vmatrix} 1 & -3 \\ 2 & 3 \end{vmatrix} + 1 \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}$

$= 1(6-3) - 1(3+6) + 1\cdot(-1-4)$

$= 1(3) - 1(9) + 1(-5)$

$= 3 - 9 - 5$

$\boxed{S_3 = -11}$

the charactric equation are

$\lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0$

by calyhoy Hamilton theorem

Every square matrix satified its owen charactric equation.

$A^3 - 6A^2 + 5A + 11I = 0$

$A^3 = A^2 \cdot A$

$A^2 = A \cdot A$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+5 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

to find A^{-1}

The charactric equation

$\lambda^3 - 6\lambda^2 + 5\lambda + 11 = 0$

(ie) $A^3 - 6A^2 + 5A + 11I = 0$

Premultiple by A^{-1}

$A^{-1}A^3 - 6A^{-1}A^2 + 5A^{-1}A + 11A^{-1}I = 0$

$A^2 - 6A - 5I + A^{-1} = 0$

$11A^{-1} = A^2 + 6A + 5I$

$$-A^2 - \begin{bmatrix} -4 & -2 & -1 \\ 3 & -8 & 14 \\ -7 & 3 & -14 \end{bmatrix}$$

$$= 6A = 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 11 & -6 & 18 \end{bmatrix}$$

$S_1 = 6$

$S_2 =$

$$5I = 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$S_1 = 6$

$S_2 = 11$

$S_3 = 6$

the characteristic equation are

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$\lambda = 1, \quad \lambda^2 - 5\lambda - 6 = 0$

$\lambda = 1, \quad (\lambda - 2)(\lambda - 3) = 0$

$\lambda = 1, \quad (\lambda - 2) = 0 \; (\lambda - 3) = 0$

$\lambda = 1, \; \lambda = 2, \; \lambda = 3$

the eigen value are $\lambda = 1, 2, 3$.

SASURIE COLLEGE OF ENGINEERING
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DEPARTMENT OF SCIENCE AND HUMANITIES

Assignment Answer Sheet

Name of the Student : R. Sriharishma

AU Register Number: 732124104098

Assignment – 02			Date of Issue:	18.11.2024	Marks	10
Course code	MA3151	Course Title	MATRICES AND CALCULUS			
Year	I	Semester/Section	I / B	Date of Submission:	02.12.24	

Q.No	Questions	CO
1	If u = $\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}= \sin 2u$.	C03
2	Find $\int(\log x)^3\, dx$ by using integral by parts.	C04

Mark Allocation

Rubrics	Marks Allocated	Marks obtained
Content Quality	6	6
Presentation Quality	2	2
Timely submission	2	2
Total marks	10	10

Saravanakumar.
02/12/24

Name and Signature of the Faculty Incharge

HoD/S&H

Find $\int (\log x)^3 \, dx$ by using integral by parts.

Soln.

Let, $u = (\log x)^3$

$$\frac{du}{dx} = 3(\log x)^2 \cdot \frac{1}{x}$$

$$du = \frac{3(\log x)^2}{x} \, dx$$

$$\int dv = \int dx$$

$$v = x$$

Integral by parts,

$$\int u\,dv = uv - \int v\,du$$

$$\int (\log x)^3 dx = (\log)^3 (x) - \int x \; \frac{3(\log x)^2}{x} dx$$

$$= x(\log x)^3 - 3\int (\log x)^2 dx \longrightarrow (1)$$

To Find $\int (\log x)^2 dx$

$u = (\log x)^2$ $\quad \int dv = \int dx$

$\frac{du}{dx} = 2\log x \left(\frac{1}{x}\right)$ $\quad v = x$

$$du = \frac{2\log x}{x} dx$$

$$\int (\log x)^2 dx = (\log x)^2 (x) - \int x \left(\frac{2\log x}{x}\right) dx$$

$$= x(\log x)^2 - 2\int \log x \, dx \longrightarrow (2)$$

To Find $\int \log x$

Let, $du = \log x$ $\quad \int dv = \int dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = x$

$$du = dx/x$$

$$\int \log x \, dx = (\log x)(x) - \int x \frac{dx}{x}$$

$$= x \log x - \int dx$$

$$= x \log x - x + c \longrightarrow (3)$$

equation (3) in equation (2)

$$\int (\log x)^2 \, dx = x(\log x)^2 - 2[x \log x - x] + c$$

$$\int (\log x)^2 \, dx = x(\log x)^2 - 2x \log x + 2x + c \longrightarrow (4)$$

equation (4) in equation (1)

$$\int (\log x)^3 \, dx = x(\log x)^3 - 3\int (\log x)^3 \, dx$$

$$= x(\log x)^3 - [3x(\log x)^2 - 2x \log x + 2x] + c$$

$$= x(\log x)^3 - 3x(\log x)^2 + 6x \log x - 6x + c$$

Evaluate $\displaystyle\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} \, dx$

Soln:-

using long division method.

$$\begin{array}{r|l}
 & 3x^2 + 1 \\
\hline
x^2 + x - 2 & 3x^4 + 3x^3 - 5x^2 + x - 1 \\
 & 3x^4 + 3x^3 - 6x^2 \\
 & (-) \quad (-) \quad (+) \\
\hline
 & x^2 + x - 1 \\
 & x^2 + x - 2 \\
 & (-) \quad (-) \quad (+) \\
\hline
 & 1 \\
\hline
\end{array}$$

$$\int \frac{3x^4 + 3x^2 - 5x^2 + x - 1}{x^2 + x - 2}\, dx = \int (3x^2 + 1)\, dx + \int \frac{1}{x^2 + x - 2}\, dx$$

$$= 3\int x^2\, dx + \int dx + \int \frac{1}{x^2 + x - 2}\, dx$$

$$= 3\left[\frac{x^3}{3}\right] + x + \int \frac{1}{x^2 + x - 2}\, dx \longrightarrow (1)$$

To Find, $\int \frac{1}{x^2 + x - 2}\, dx$,

Here, $x^2 + x - 2$

$$= x^2 + x + \frac{1}{4} - \frac{9}{4}.$$

$$= (x + 1/2)^2 - (3/2)^2$$

$$\int \frac{1}{x^2 + x - 2}\, dx = \int \frac{1}{(x + 1/2)^2 - (3/2)^2}\, dx$$

WKT,

$$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left|\frac{x - a}{x + a}\right| + c$$

$$= \frac{1}{2(3/2)} \log \left|\frac{x + 1/2 - 3/2}{x + 1/2 + 3/2}\right| + c$$

$$= \frac{1}{3} \log \left|\frac{x - 2/2}{x + 4/2}\right| + c$$

$$= \frac{1}{3} \log \left|\frac{x - 1}{x + 2}\right| + c \longrightarrow (2)$$

Sub (2) in eq (1)

$$\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2}\, dx = x^3 + x + \frac{1}{3} \log \left|\frac{x - 1}{x + 2}\right| + c$$

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(x + 1/2)^2 = x^2 + 2x(1/2) + 1/4 \Longrightarrow x^2 + x + 1/4 \quad a^2 = x^2 \quad \boxed{a = x}$$

$$2ab = x$$

$$2x\,b = x$$

$$2b = 1$$

$$b = 1/x.$$

$u = \sin^{-1}\left[\frac{x^3+y^3}{x+y}\right]$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$

Soln :-

$$u = \tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$$

Let, $\tan u = z$

$$z = \left(\frac{x^3+y^3}{x-y}\right)$$

$$= \frac{x^3\left(1+y^3/x^3\right)}{x\left(1-y/x\right)}$$

$$= \frac{x^2\left(1+y^3/x^3\right)}{\left(1-y/x\right)}$$

by Euler's theorem,

z is an homogeneous function of degree 2.

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z \longrightarrow (1)$$

$$z = \tan u$$

$$\frac{\partial z}{\partial x} = \sec^2 u\,\frac{\partial u}{\partial x}$$

$$x\frac{\partial z}{\partial x} = \sec^2 u \quad x\frac{\partial u}{\partial x} \longrightarrow (2)$$

$$\frac{\partial z}{\partial y} = \sec^2 u \frac{\partial u}{\partial y}$$

$$y \frac{\partial z}{\partial y} = \sec^2 u \; y \frac{\partial u}{\partial y} \longrightarrow (5)$$

Add eq (5), (2)

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \sec^2 u \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right)$$

From equation,

$$\frac{2 \tan u}{\sec^2 u} = x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

$$2 \times \frac{\sin u}{\cos u} \times \cos^2 u = x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

$$2 \sin u \cos u = x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$$

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

$2 \sin A \cos A = \sin 2A.$

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UNIT-1 MATRICES

PART – A

1. State Cayley- Hamilton theorem.
2. Find the sum and product of the Eigenvalues of the matrix $A = \begin{pmatrix} 3 & 8 & 6 \\ 8 & 4 & 2 \\ 6 & 2 & 5 \end{pmatrix}$
3. Find the sum and product of the Eigenvalues of the matrix $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$
4. The Eigen value of a matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0, what is the third Eigen value? And find the product of the Eigen value?
5. Find the sum and product of all the Eigenvalues of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
6. If 2 and 3 are the two eigenvalues of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$ then find the value of b.
7. The product of two Eigenvalues of the $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16.find the third Eigenvalue.
8. Find the Eigenvalues of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.
9. Find the Eigenvalues of 3A+2I, where A= $\begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix}$.
10. If λ is an Eigen value of a matrix A, then λ^{-1} is the Eigen value of A^{-1}.
11. If λ is an Eigen value of a matrix A, then λ^2 is the Eigen value of A^2.
12. Prove that the Eigen value of a orthogonal matrix are of unit modulus.
13. If the Eigen value of the matrix 3x3 are 2,3,1 then find the Eigen value of adjoint of A.
14. If 2,-1,-3 are the Eigen value of the matrix A,then find the Eigen value of $A^2 - 2I$.
15. If the sum of two Eigen values and trace of a 3x3 matrix A are equal, find the value of |A|.

16. Prove that $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2zx = 0$ is indefinite.

17. Give the nature of a quadratic from whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

18. What is the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables?

19. Discuss the nature of the quadratic form $2x^2 + 3y^2 + 2z^2 + 2xy$.

20. Write down the matrix corresponding to the quadratic form $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$.

PART-B

CHAPTER-1.1 (8-MARKS)

1. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

2. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$

3. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$

4. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

5. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

6. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

7. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

8. Find the Eigen values and Eigen vectors for the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

CHAPTER-1.2 (8-MARKS)

1. Verify the Cayley-Hamilton theorem and also find A^{-1} for the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

2. Verify the Cayley-Hamilton theorem and also find A^{-1} for the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -3 & 1 \\ 2 & 1 & -2 \end{bmatrix}$

3. Verify the Cayley-Hamilton theorem and also find A^{-1} for the matrix $\begin{bmatrix} -3 & 2 & 1 \\ 3 & -1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

4. Verify the Cayley-Hamilton theorem and also find A^{-1} for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

5. Verify the Cayley-Hamilton theorem and also find A^{-1} for the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$

6. Verify the Cayley-Hamilton theorem and also find A^4 for the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$

7. **Use Cayley-Hamilton theorem to find the value of**

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I \quad \text{Where } A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

8. Verify the Cayley-Hamilton theorem and also find A^4 for the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

CHAPTER-1.3 (16-MARKS)

1. Reduce the quadratic form into the canonical by using orthogonal transform $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ and also find Rank, signature, Index

2. Reduce the quadratic form into the canonical by using orthogonal transform $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ and also find Rank, signature, Index

3. Reduce the quadratic form into the canonical by using orthogonal transform $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ and also discuss the nature

4. Reduce the quadratic form into the canonical by using orthogonal transform $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_2x_3 + 2x_3x_1$ and also discuss the nature

5. Reduce the quadratic form into the canonical by using orthogonal transform $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ and also find Rank, signature, Index

6. Reduce the quadratic form into the canonical by using orthogonal transform $3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz$. and also discuss the nature

7. Reduce the quadratic form into the canonical by using orthogonal transform $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$. and also find Rank, signature, Index

8. Reduce the quadratic form into the canonical by using orthogonal transform $x_1^2 + 2x_2^2 + x_3^2 - 12x_1x_2 + 2x_2x_3$ and also find Rank, signature, Index

Reduce the quadratic form into the canonical by using orthogonal transform $2x^2 + y^2 + z^2 + 2xy - 2xz - 4yz$ and also find Rank, signature, Index

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UNIT- 2 DIFFERENTIAL CALCULUS

PART – A

1. Find the domain and range $f(x) = 3x-2$.
2. Sketch the graph of the absolute value function $f(x)=|x|$
3. Prove that $\lim_{x\to 0}|x| = 0$.
4. If $x^2 + y^2 = 25$, then find $\frac{dy}{dx}$.
5. Find the derivative $y = (x^3-1)^{100}$.
6. Find the domain and range $y = x^2$.
7. Sketch the graph of function $|x| = \begin{cases} x, & if\, x > 0 \\ -x, & if\, x < 0 \end{cases}$
8. Find the $\lim_{x\to 3^+}\left(\frac{2x}{x-3}\right)$.
9. Prove that $\lim_{x\to 0}\left(\frac{|x|}{x}\right)$
10. Define derivative of a function $f(x)$.
11. Evaluate $\lim_{x\to 1}\left(\frac{x^4-1}{x^3-1}\right)$, if it exists.
12. Find the derivative of the function $f(x) = \sqrt[3]{1+tanx}$.
13. Sketch the graph of the function $\begin{cases} 1+x, x<-1 \\ x^2, -1\le x\le 1 \\ 2-x, x\ge 1 \end{cases}$ and use it to determine the value of "a" for which $\lim_{x\to a} f(x)$ exists? **(Jan-18)**
14. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents? If so where? **(Jan-18)**
15. State Rolle's Theorem and verify the Rolle's theorem for $f(x) = x^3 + 5x^2 - 6x$ on the interv l (0, 1).
16. State Mean value theorem
17. Find the critical numbers for $f'(x) = \frac{x^2(x-1)}{x+2}, x \neq -2$
18. Define concavity and point of inflection.

19. Define maxima and minima of one variable and write the conditions.

20. Find the tangent line and normal line to the given curve $y = 2xe^x$ at (0, 0).

21. Find the domain of $f(x) = \sqrt{3-x} - \sqrt{2+x}$ (Nov 2018)

22. Evaluate $\lim_{t\to 1} \frac{t^4-1}{t^3-1}$ (Nov 2018)

23. check whether $\lim_{t\to -1} \frac{3x+9}{|x+3|}$ exist. (APR 19)

24. Find the critical points of $y = 5x^3 - 6x$ (APR 19)

PART – B

Limits

1. Find the value of $\lim_{x\to 0} \frac{\sin x}{x}$

2. Find the domain of the functions a) $y = x^2$, b) $f(x) = \sqrt{x-2}$, c) $g(x) = \frac{1}{x^2-x}$

3. Find $\lim_{x\to 0}\left(\frac{1}{x^2}\right)$.

4. Evaluate $\lim_{x\to\frac{\pi}{2}} \frac{1+\cos 2x}{(\pi-2x)^2}$.

5. Find the limit of the function $\lim_{x\to 0} \frac{e^{5x}-1}{x}$, given numbers $x = \pm 0.5$, ± 0.1 , ± 0.01 , ± 0.001 , ± 0.001 (correct 6 decimal places) (Nov 2018) .

Continuity

1. Show that the function $f(x) = 1-\sqrt{1-x^2}$ is continuous on the interval $[-1, 1]$.

2. Find an equation of the tangent line to the parabola $y = x^2$ at the point (1,1) .

3. Find an equation of the tangent line to the hyperbola $y = \frac{3}{x}$ at the point (3,1) .

4. If $(x) = \sqrt{x}$, find the equation for $f'(x)$.

5. Determine whether $f'(0)$ exist or not for the given function

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

6. Where is the function $f(x) = |x|$ is differentiable?

7. Find $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.

8. Find the value of $\lim_{x\to 1}\left(\frac{x-1}{x^2-1}\right)$

9. For what value of the constant "c" is the function "f" continuous on $(-\infty,\infty)$,

$$f(x)=\begin{cases} cx^2+2x, x<2 \\ x^3-cx, x\geq 2 \end{cases}$$

10. For what value of the constant "b" is the function "f" continuous on $(-\infty,\infty)$,

if $f(x)=\begin{cases} bx^2-2x & if \ \ x<2 \\ x^3-bx & if \ \ x\geq 2 \end{cases}$ (Apr 19)

Differentiability

1. If the function f(x) is differentiable at a, then f(x) is continuous at a .

2. Find an equation of the tangent line to the hyperbola $y=\frac{3}{x}$ at the point (3,1).

3. Find an equation of the tangent line to the parabola $y=x^2-8x+9$ at the point (3,-6) .

4. Where is the function $f(x)=|x|$ is differentiable?

5. Show that $\lim\limits_{x\to 0} x^2 \sin\frac{1}{x}=0$.

6. Show that the sum of x and x intercepts of any tangent line to the curve $\sqrt{x}+\sqrt{y}=\sqrt{c}$ is equal to c.

7. Verify that the function $f(x)=5-12x+3x^2$ satisfies the Rolle's theorem on the interval [1.3].

8. Find the local maximum and local minimum values of the function g(x)=x+2sinx.

9. Find the absolute maximum and minimum values of the function

$f(x)=x^3-3x^2+1, \quad \frac{-1}{2}\leq x\leq 4.$

10. Find the local maximum and local minimum values of the function

$f(x)=2x^3+3x^2-36x.$

11. Find the values of a and b such that the function $f(x)=\begin{cases} x+2, \ x<2 \\ ax^2-bx+3, \ 2\leq x<3 \\ 2x-a+b, \ x\geq 3 \end{cases}$

is continuous everywhere.

12. Find the derivative of the function $f(x)=\frac{1}{\sqrt{x}}$ using the definition of derivative. 0

13. Find the values of a and b such that the function $f(x)=\begin{cases} \frac{x^3-8}{x-2}, \ x<2 \\ ax^2-bx+3, \ 2\leq x\leq 3 \\ 2x-a+b, \ x\geq 3 \end{cases}$ (Nov 2018)

14. find the derivative of $f(x)=\cos^{-1}\left[\frac{b+a\cos x}{a+b\cos x}\right]$ (Nov 2018)

15. find y' for $\cos(xy)=1+\sin y$ (Nov 2018)

16. Find $\frac{dy}{dx}$ if $y=x^2e^{2x}(x^2+1)^4$ (Apr 19)

Maxima and minima

1. Find the maximum and minimum values of $f(x)=\frac{x^2+x+1}{x^2-x+1}$.

2. Find the local maximum and minimum values of $f(x) = \sqrt{x} - \sqrt[4]{x}$ using both the first and second derivative tests. **(Jan-18)**

3. Find y'' if $x^4 + y^4 = 6$ **(Jan-18)**

4. Find the tangent line to the equation $x^3 + y^3 = 6xy$ at the point (3, 3) and at what point the tangent line horizontal in the first quadrant. **(Jan-18)**

5. For the function $f(x) = 2 + 2x^2 - x^4$ find the intervals of increase or decrease , local maximum and minimum values, concavity and inflection points **(Nov 2018)**

6. For the function $f(x) = 2x^3 + 3x^2 - 36x$ find the intervals of increase or decrease, local maximum and minimum values. **(Apr 19)**

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UNIT- 3 FUNCTIONS OF SEVERAL VARIABLE

PART – A

1. If $u = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = ?$
2. If $x^y + y^x = 1, then\ find\ \frac{dy}{dx} = ?$
3. If $u = x^2 + y^2$ and $x = at^2, y = 2at$, find $\frac{du}{dt}$.
4. If $x = r\cos\theta, y = r\sin\theta$, then find $\frac{\partial(x,y)}{\partial(r,\theta)}$.
5. If $u = \frac{y^2}{x}, v = \frac{x^2}{y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.
6. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
7. If $u = \frac{y}{z} + \frac{z}{x}$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = ?$
8. If $u = x+y$ and $y = uv$, find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$.
9. Write down Taylor's formula.
10. Find dy/dx when $x^2+y^2 = xy$.
11. Are $u = x + y$ and $v = x - y$ functionally independent? Justify the claim.
12. If $x = r\cos\theta, y = r\sin\theta$, then find $\frac{\partial r}{\partial x}$ (Jan-18)
13. If $x = uv, y = \frac{u}{v}$, find $\frac{\partial(x,y)}{\partial(u,v)}$. (Jan-18)
14. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ if $u = y^x$
15. If $u = (x-y)(y-z)(z-x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
16. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
17. Find $\frac{du}{dt}$ if $u = \frac{x}{y}$ where $x = e^t$, $y = \log t$
18. Find $\frac{du}{dx}$ if $u = \sin(x^2 + y^2)$, where $3x^2 + y^3 = 4$

19. If $x = u^2 - v^2$, $y = 2uv$ evaluate the Jacobian of **x, y** with respect to **u, v** **(Apr19)**

20. If $x^2 + y^2 = 1$, then find $\frac{dy}{dx}$.

21. Find $\frac{dy}{dx}$ if $x^y + y^x = c$, where c is a constant **(Nov18)**

22. State the properties of jacobians **(Nov18)**

23. Find $\frac{du}{dt}$ in trems of t, if $x^3 + y^3 = u$ where $x = at^2, y = 2at$ **(Apr19)**

PART – B

Implicit functions

1. If $u = \sin^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$.

2. If $g = \Psi(u, v)$ *where* $u = x^2 - y^2$ *and* $v = 2xy$,

   show that $\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2 + y^2)\left[\frac{\partial^2 \Psi}{\partial u^2} + \frac{\partial^2 \Psi}{\partial v^2}\right]$

3. If $u = f(x - y, y - z, z - x)$ *then show that* $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

4. If $\emptyset = \emptyset(u, v)$ *where* $u = e^x \cos y, v = e^x \sin y$

   Show that $\frac{\partial^2 \emptyset}{\partial x^2} + \frac{\partial^2 \emptyset}{\partial y^2} = (u^2 + v^2)\left[\frac{\partial^2 \emptyset}{\partial u^2} + \frac{\partial^2 \emptyset}{\partial v^2}\right]$.

5. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

6. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, then find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$, using Euler's theorem.

7. If $z = f(x,y)$, where $x = e^u \cos v$ *and* $y = e^u \sin v$ then show that $x\frac{\partial z}{\partial v} + y\frac{\partial z}{\partial u} = e^{2u}\frac{\partial z}{\partial y}$.

8. If $u = (x^2 + y^2 + z^2)^{-1/2}$ then find the value of $u_{xx} + u_{yy} + u_{zz}$. **(Jan-18)**

9. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$ *find* $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z}$. **(Nov-18)**

10. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then find $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z}$ **(Apr19)**

Maxima and minima

11. Find the Maximum and Minimum of $f(x,y) = x^2 - xy + y^2 - 2x + y$.
12. Find the Maximum and Minimum of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
13. Find the Maximum and Minimum of $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **(Nov-18)**
14. Examine $f(x,y) = x^3 - 15y^2 - 15x^2 + 3xy^2 + 72x$ for extreme values. **(Apr19)** 3 86.

Jacobian

15. *If* $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_3x_1}{x_2}, y_3 = \frac{x_1x_2}{x_3}$ *then find* $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$.

16. If $x + y + z = u, y + z = uv, z = uvw$, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.

17. Find the maximum and minimum values of $f(x,y) = 3x^2 - y^2 + x^3$. **(Jan-18)**

Lagrange's multiplier method

18. A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box requiring, the least material for the construction.

19. The temperature at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

20. Find the shortest and the longest distances from the point (1,2,-1) to the spher $x^2 + y^2 + z^2 = 24$. **(Apr19)**

21. Find the dimensions of the rectangular box without a top of maximum capacity, whose surface area is 108 sq. cm. **(Jan-18)**

22. Find the volume of the greatest rectangular parallelepiped that can be Inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, using Lagrange's method.

23. Find the shortest distances from the point $(1, 2, 0)$ to the cone $x^2 + y^2 = z^2$. **(Nov-18)**

Taylor's series method

24. Expand $f(x, y) = e^x \cos y$ at $\left(0, \frac{\pi}{2}\right)$ upto 3rd term using Taylor's series 3.62

25. Obtain the Taylor's series expansion of $x^3 + y^3 + xy^2$ in terms of power of (x-1) and (y-2) upto third degree terms. 3.72 **(Jan-18)**

26. Find the taylor's series of function $f(x) = \sqrt{1 + x + y^2}$ in powers $(x - 1)$ *and* y upto second degree terms. **(Nov-18)**

27. Expand Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in terms of power of $(x + 2)$ and $(y - 1)$ upto third degree terms. **(Apr19)**

MATHS CHAP-4

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PART-A

1. Evaluate $\int \theta \cos\theta \, d\theta$ using integration by parts. Refer (Eg:1) 4.10
2. Find the value of $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$ Same as (3)
3. Find the value of $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$ Refer 4.25 [illegible]
4. Evaluate $\int \frac{\cos\theta}{\sin^3\theta} \, d\theta$ by the method of substitution. Pg 4.55 [illegible]
5. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x^3 + x^4 \tan x) \, dx$ 4.64 (Eg.26)
6. Given that $\int_0^{10} f(x)dx = 17$ and $\int_0^{8} f(x)dx = 12$ then find $\int_8^{10} f(x)dx$ pg no
7. If f is continuous and $\int_0^{4} f(x) \, dx = 10$ find $\int_0^{2} f(2x)dx$ ans
8. Prove that $\int_0^{a} f(x)dx = \int_0^{a} f(a-x)dx$. ans
9. Evaluate $\int \frac{\tan x}{\sec x + \tan x} \, dx$
10. Determine whether the integral which is Convergent or Divergent

    (a) $\int_0^{\infty} \frac{1}{x} \, dx$ (b) $\int_0^{\infty} \frac{dx}{x^2+4}$ (c) $\int_0^{\infty} e^x \, dx$ (d) $\int_4^{\infty} \frac{1}{\sqrt{x}} \, dx$ (e) $\int_3^{\infty} \frac{dx}{\sqrt[5]{x^2-x-3}}$ → Pg.No 4.20

    (f) $\int_1^{\infty} \frac{dx}{e^x+x^2}$

    4th sum ↓ pg no [illegible] (F)

PART- B

1. Evaluate $\int (\log x)^3 \, dx$ by using integration by parts.
2. Evaluate $\int x^2 e^x \, dx$ by using integration by parts.
3. Using by integration by parts $\int \frac{(\log x)^2}{x^2} \, dx$ 4.81 (Eg [illegible])
4. Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x \, dx$ and hence find the value of $\int_0^{1} \frac{\sin^{-1} x}{x} \, dx$
5. Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) \, dx = \frac{\pi}{8} \log 2$
6. Establish a reduction formula for $I_n = \int \sin^n x \, dx$ Hence find $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$

7. Establish a reduction formula for $I_n = \int cos^n x\, dx$ Hence find $\int_0^{\frac{\pi}{2}} cos^n x\, dx$

8. Evaluate $\int_0^{\infty} e^{-ax} sinbx\, dx$ $(\alpha > 0)$using integration by parts.

9. Evaluate $\int_0^{\infty} e^{-ax} cosbx\, dx$ $(\alpha > 0)$using integration by parts

10. Evaluate $\int \frac{x+\sin x}{1+cosx} dx$

11. Evaluate $\int \frac{sec^2 x}{tan^2 x+3\, tanx+2}\, dx$

12. Evaluate $\int_0^{\frac{\pi}{2}} \frac{sinx\, cosx}{cos^2 x+3cosx+2}\, dx$

13. Evaluate $\int \frac{10}{(x-1)(x^2+9)}\, dx$ using partial fraction method.

14. Evaluate (a) $\int \frac{dx}{\sqrt{3x-x^2-2}}$ (b) $\int \frac{dx}{\sqrt{3x^2+x-2}}$

15. Evaluate $\int \frac{2x+3}{x^2+x+1}\, dx$

16. Evaluate $\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\, dx$ by using method of partial fraction.

17. Evaluate $\int \frac{x+4}{6x-7-x^2}\, dx$

18. Evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (tan^2 x sec^2 x)\, dx$

19. Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)}\, dx$ by applying partial fraction on the integral.

20. Evaluate $\int \frac{3x^4+3x^2-5x^2+x-1}{x^2+x-2}\, dx$

$\frac{d}{dx} \sin x = \cos x$ $\qquad \int \cos x\, dx = \sin x + c$

$\frac{d}{dx} \cos x = -\sin x$ $\qquad \int \sin x\, dx = -\cos x + c$

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SASURIE

SASURIE INSTITUTIONS

COLLEGE OF ENGINEERING

Vijayamangalam, Tiruppur

UNIT 5

PART- A

1. Evaluate $\int_0^1 \int_0^x dx\,dy$.
2. Evaluate $\int_1^2 \int_1^2 xy^2 dx\,dy$.
3. Evaluate $\int_1^2 \int_2^5 xy\,dx\,dy$.
4. Find the value of $\int_0^\infty \int_0^y \frac{e^{-y}}{y} dx\,dy$.
5. Find the value of $\int_0^2 \int_0^{x^2} e^{y/x}\,dx\,dy$.
6. (a) Find the value of $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\,dx\,dy$

   (b) Find the value of $\int_0^1 \int_1^2 x(x+y)\,dx\,dy$.
7. Find the value of $\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} r^2\,dr\,d\theta$.
8. Find the value of $\int_1^a \int_1^b \frac{1}{xy} dx\,dy$.
9. Change of order of integration

   (a) $\int_0^2 \int_0^x f(x,y)dx\,dy$. (b) $\int_0^1 \int_1^{\sqrt{4-y}} f(x,y)dx\,dy$.
10. Sketch the region of integration in $\int_0^1 \int_x^1 f(x,y)dx\,dy$.
11. Draw the rough sketch for the region of integration $\iint f(x,y)\,dx\,dy$, where the region is the triangle in xy-plane bounded by x- axis y=x and the line $x = \pi/2$.

12. For the limit of f the integration $\iint_R f(x,y)dx\,dy$, where R is the region bounded by the lines x = 0, y = 0 and x+y = 2.

13. Evaluate $\int_0^1\int_0^2\int_0^3 (xy^2 z)\,dx\,dy.$

14. Describe the solid region whose volume is given by the following triple integral $\int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{z=0}^{1} dz\,dy\,dx$. (do not evaluate the integral).

15. Transfer the double integral $\int_0^2\int_y^2 \frac{x\,dx\,dy}{x^2+y^2}$ into polar coordinates.

PART – B

1) Change of order of integration for the given integrals and also evaluate it

a) $\int_0^a\int_0^{2\sqrt{ax}} x^2\,dx\,dy$ b) $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{2a}} xy\,dx\,dy$ c) $\int_0^a\int_x^a (x^2+y^2)\,dx\,dy$.

2) Find the area between the curves $y^2 = 4x$ and $x^2 = 4y$. (OR) Change of order of integration on $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dx\,dy$ and hence evaluate.

3) Change the order of integration in

(a) $\int_0^\infty\int_0^y ye^{-y^2/x}\,dx\,dy.$ (b) $\int_0^\infty\int_0^x xe^{-x^2/y}\,dx\,dy$ and evaluate it.

4) Change of order of integration for the given integrals $\int_0^a\int_{t^2/a}^{2a-x} xy\,dx\,dy$ and also evaluate it.

5) By changing polar coordinates , evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$.

6) Evaluate $(a)\int_0^2\int_0^{\sqrt{2x-x^2}}(x^2+y^2)\,dx\,dy$. (b) $\int_0^a\int_0^{\sqrt{(a^2-x^2}}(x^2+y^2)\,dx\,dy$ by changing into polar coordinates.

7) Evaluate by changing into polar coordinates $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy$.

8) Using a double integral, the area of the cardioid r = a (1+ cosθ).

9) Find by double integral the area between the parabola $y^2 = 4ax$ and the line y = x. 5 S2

10) Find the area of the region R enclosed by the parabola y = x^2 and the line y = x +2

11) Evaluate $\iint (x^2y + y^2x)dx\, dy$ over the area between y = x^2 and y = x.

12) Evaluate $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} (z) dz\, dy\, dx.$

13) Find the value of $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{1-x^2-y^2-z^2}}$. (OR)

Find the value of $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{a^2-x^2-y^2-z^2}}$. (OR)

✓Evaluate $\iiint \frac{dz\, dy\, dx}{\sqrt{a^2-x^2-y^2-z^2}}$ over the firstn octant of the sphere

$x^2 + y^2 + z^2 = a^2$

14) Evaluate $\int_0^{2a} \int_0^x \int_y^x (xyz) dz\, dy\, dx.$

15) Find the volume of the sphere of radius 'a'. (OR)

Find the volume of sphere $x^2 + y^2 + z^2 = a^2$ using triple integral. (OR)

Find the volume of sphere (a) $x^2 + y^2 + z^2 = 9$ (b) $x^2 + y^2 + z^2 = 16$

(c) $x^2 + y^2 + z^2 = 25$

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30234

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

First Semester

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two eigen values of the matrix $A=\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are equal to 1 each, find the eigen value of A^{-1}.

2. Write the uses of Cayley-Hamilton Theorem.

3. If $y = x\log\left(\frac{x-1}{x+1}\right)$, then find $\frac{dy}{dx}$.

4. Find the point of inflection of $f(x)=x^3-9x^2+7x-6$.

5. Write Euler's theorem on homogeneous functions.

6. If $x=r\cos\theta$, $y=r\sin\theta$, find $\frac{\partial(x,y)}{\partial(r,\theta)}$.

7. Evaluate $\int \theta\cos\theta\, d\theta$ using integration by parts.

8. Find the value of $\int_0^{\pi/2} \sin^6 x\, dx$.

9. Evaluate $\int_0^1\int_0^x dy\, dx$.

10. Transform the double integral $\int_0^2\int_y^2 \frac{x\,dx\,dy}{x^2+y^2}$ into polar coordinates.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x_1x_2 - 2x_2x_3 + 2x_3x_1$ into the canonical form and hence find its nature. (16)

12. (a) (i) Find the values of a and b that make f continuous on $(-\infty, \infty)$ if

$$f(x)=\begin{cases}\dfrac{x^3-8}{x-2}, & if\ x<2\\ ax^2-bx+3, & if\ 2\le x<3\\ 2x-a+b, & if\ x\ge 3\end{cases}$$ (8)

(ii) Find $\dfrac{dy}{dx}$ if $y=x^2e^{2x}(x^2+1)^4$. (4)

(iii) If $x^y=y^x$, Prove that $\dfrac{dy}{dx}=\dfrac{y(y-x\log y)}{x(x-y\log x)}$ using implicit differentiation. (4)

Or

(b) (i) Show that $\sin x(1+\cos x)$ is maximum when $x=\pi/3$. (6)

(ii) A window has the form of a rectangle surmounted by a semicircle. If the perimeter is 40 ft., find its dimensions so that greatest amount of light may be admitted. (10)

13. (a) (i) Given the transformations $u=e^x\cos y$ and $v=e^x\sin y$ and that f is a function of u and v and also of x and y, prove that $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}=(u^2+v^2)\left(\dfrac{\partial^2 f}{\partial u^2}+\dfrac{\partial^2 f}{\partial v^2}\right)$. (8)

(ii) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y)x^4+y^4-2x^2+4xy-2y^2$. (8)

(ii) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction. (8)

14. (a) (i) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)}dx$ by applying partial fraction on the integrand; (6)

(ii) Evaluate $\int_0^{\pi/2} \log \sin x\, dx$ and hence find the value of $\int_0^1 \frac{\sin^{-1} x}{x} dx$. (10)

Or

(b) (i) Evaluate $\int \frac{\sqrt{9-x^2}}{x^2} dx$ using trigonometric substitution. (6)

(ii) Determine whether the integral $\int_1^{\infty} \frac{1}{x} dx$ is convergent or divergent. (4)

(iii) Find the volume of the reel shaped solid formed by the revolution about the y-axis, of the part of the parabola $y^2 = 4ax$ cut off by its latusrectum. (6)

15. (a) (i) Find the area between the curves $y^2 = 4x$ and $x^2 = 4y$. (8)

(ii) Change the order of integration in $\int_0^{\infty}\int_0^{y} y e^{-y^2/x} dx\, dy$ and then evaluate it. (8)

Or

(b) (i) Find the volume of the sphere of radius 'a'. (8)

(ii) Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 51315

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except B.E. Marine Engineering))

(Also Common to PTMA 3151-Matrices and calculus for B.E. (Part-Time)
First Semester-All Branches-Regulations 2023)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is an eigenvalue of a matrix A, then prove that λ^2 is an eigenvalue of A^2.

2. If $x = [-1, 0, 1]^T$ is the eigenvector of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, then find the corresponding eigen value.

3. Sketch the graph of the function $f(x) = 2.0 - 0.4x$ and find the domain of the function.

4. Differentiate $y = x\tan\left(\sqrt{x}\right)$ with respect to x.

5. Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.

6. If $u = x - y$, $v = y - z$, $w = z - x$, then find the Jacobian $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$.

7. What is wrong with the equation $\int_{-2}^{1}\left[\dfrac{1}{x^4}\right]dx = \int_{-2}^{1}\left[x^{-4}\right]dx = \left[\dfrac{x^{-3}}{-3}\right]_{-2}^{1} = -\dfrac{3}{8}$.

8. Evaluate $\int_{-1}^{1}\left[\dfrac{\tan x}{1+x^2+x^4}\right]dx$ by using the concept of odd and even functions.

9. Evaluate $\int_1^2 \int_0^{x^2} [x] dy dx$.

10. Write the integral equation for the regions $x \geq 0, y \geq 0, z \geq 0$, $x^2 + y^2 + z^2 \leq 1$ by triple integration.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ to a canonical form by orthogonal reduction. (16)

12. (a) (i) Find the value of $\lim_{x \to 2} \left[\dfrac{x^2 - 2}{x^3 - 3x + 5} \right]^2$. (6)

(ii) Find the local maximum and minimum values of the function $f(x) = x + 2\sin x$ in the interval $0 \leq x \leq 2\pi$. (10)

Or

(b) (i) Find an equation of the tangent line to the curve $y = \dfrac{e^x}{(1 + x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = \log\left[x^2 + x + 1\right]$ in the interval $[-1, 1]$. (8)

13. (a) (i) If $u = \log\left[x^2 + y^2 + z^2\right]$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$? (8)

(ii) The temperature at any point (x, y, z) in space is given by $T = 400xyz^2$. Find the maximum temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (8)

Or

(b) (i) Expand $f(x,y) = e^{x+y}$ about the point $(0,0)$ in powers of x and y upto third degree terms by using Taylor's series. (8)

(ii) Find the maxima and minima for the given function $f(x,y) = x^3y^2[1 - x - y]$. (8)

14. (a) (i) Evaluate $\int x^2 e^x \, dx$ by using integration by parts. (8)

(ii) Evaluate the integral $\int \sin^4 x \, dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{a^2 - x^2}\, dx$. (8)

(ii) Evaluate $\int \frac{1}{(x^2 - a^2)} dx$ by using partial fraction. (8)

15. (a) (i) Evaluate $\int_0^{\pi/2} \int_0^{\sin\theta} [r] \, d\theta \, dr$. (8)

(ii) Change the order of integration in

$\int_0^a \int_x^a \left[x^2 + y^2\right] dy \, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\iint [xy]\, dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 3^2$ by using triple integration. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21272

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigenvalues of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.

2. State Cayley-Hamilton theorem.

3. Sketch the graph of the function $f(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 0 \\ 2-x & \text{if } 0 < x \le 2 \end{cases}$.

4. The equation of motion of a particle is given by $s = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t in seconds. Find the velocity and acceleration as functions of time.

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. Write any two properties of Jacobians.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x \, dx$.

8. Prove that the integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent.

9. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$.

10. Find the area of a circle $x^2+y^2=a^2$ using polar coordinates in double integrals.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{pmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^{-1} if $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let $f(x)=\begin{cases}\sqrt{-x} & \text{if } x<0\\ 3-x & \text{if } 0\le x\le 3\\ (x-3)^2 & \text{if } x>3\end{cases}$. Evaluate each of the following limits, if they exist.

(1) $\lim_{x\to 0^-} f(x)$

(2) $\lim_{x\to 0^+} f(x)$

(3) $\lim_{x\to 3^-} f(x)$

(4) $\lim_{x\to 3^+} f(x)$

(5) $\lim_{x\to 0} f(x)$

(6) $\lim_{x\to 3} f(x)$

Also, find where $f(x)$ is continuous. (8)

(ii) Find the n^{th} derivative of $f(x)=xe^x$. (4)

(iii) Differentiate $F(t)=\dfrac{t^2}{\sqrt{t^3+1}}$. (4)

Or

(b) (i) Use logarithmic differentiation to differentiate $y = \dfrac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$. (8)

(ii) Discuss the curve $f(x) = x^4 - 4x^3$ for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right) \quad (8)$$

(ii) Expand $e^x \cos y$ in a series of powers of x and y as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y) = x^3 + y^3 - 12x - 3y + 20$. (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for $\int e^{ax} \sin^n x \, dx$. (8)

(ii) Integrate the following : $\displaystyle\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$. (8)

Or

(b) (i) Evaluate $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\, dx$. (8)

(ii) Find the centre of mass of a semicircular plate of radius r. (8)

15. (a) (i) Change the order of integration in $\displaystyle\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy \, dy \, dx$ and then evaluate it. (8)

(ii) Find the area enclosed by the curves $y = 2x - x^2$ and $x - y = 0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 70132

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The eigenvalues and the corresponding eigenvectors of a 2 × 2 matrix is given by $\lambda_1 = 8$; $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = 4$; $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find the corresponding matrix.

2. Determine the nature, index and signature of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$.

3. For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}.$$

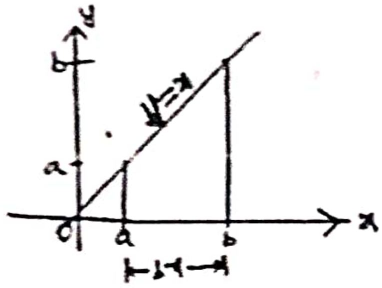
4. Find the slope of the circle $x^2 + y^2 = 25$ at $(3, -4)$.

5. Find $\frac{\partial^2 w}{\partial x \partial y}$, if $w = xy + \frac{e^y}{y^2+1}$.

6. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$ and $y = r + s$.

7. Evaluate $\int \frac{\tan x}{\sec x + \tan x} dx$.

8. Find the area of the region shown in the diagram given below, bounded between $x=a$ and $x=b$.



9. Sketch the region of integration in $\int_0^1\int_x^1 f(x,y)\,dy\,dx$.

10. Change the Cartesian integral $\int_0^6\int_0^y x\,dx\,dy$ into an equivalent polar integral.

PART B — (5 × 16 = 80 marks)

11. (a) Obtain an orthogonal transformation which will transform the quadratic form $Q=2x_2x_3+2x_3x_1+2x_1x_2$ to canonical form.

Or

(b) An elastic membrane in the x_1x_2-plane with boundary circle $x_1^2+x_2^2=1$ is stretched so that a point $P=(x_1,x_2)$ goes over a point $Q=(y_1,y_2)$ given by $y_1=5x_1+3x_2$ and $y_2=3x_1+5x_2$. Find the principal directions that is, the directions of the position vector x of P for which the direction of the position vector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation?

12. (a) (i) Find y' if $x^4+y^4=16$. (8)

(ii) Differentiate $y=(2x+1)^5(x^3-x+1)^4$. (8)

Or

(b) Find the intervals on which $f(x)=-x^3+12x+5;\ -3\le x\le 3$ is increasing and decreasing. Where does the function assume extreme values? What are those values?

13. (a) Find the maximum and minimum values of the function $f(x,y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$.

Or

(b) Find the Taylor series expansion of the function $f(x,y) = \sin x \sin y$ near the origin.

14. (a) (i) Evaluate $\int_0^{\infty} e^{-ax} \sin bx dx$, for $a > 0$. (8)

(ii) Integrate $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$. (8)

Or

(b) (i) Evaluate $\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2} dx$. (8)

(ii) Integrate $\int x\sqrt{1 + x - x^2} dx$. (8)

15. (a) (i) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$ and hence evaluate. (8)

(ii) Find the area of the region inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$. (8)

Or

(b) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. (16)

Reg. No. :

Question Paper Code : 41520

B.E./B.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to All Branches (Except : Marine Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 2, −1, −3 are the eigenvalues of a matrix "A", then find the eigenvalues of the matrix $A^2 - 2I$.

2. Write down the matrix for the following quadratic form:

   $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1 x_2 - 6x_1 x_3 + 6x_2 x_3$.

3. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$.

4. Evaluate the limit $\lim_{x \to 1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$.

5. If $u = x^3 + y^3$ where $x = a\cos t$ and $y = b\sin t$ then find $\dfrac{du}{dt}$.

6. If $u = \dfrac{2x - y}{2}$ and $v = \dfrac{y}{z}$ then find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

7. Given that $\int_0^{10} f(x)\,dx = 17$ and $\int_0^{8} f(x)\,dx = 12$ then find $\int_8^{10} f(x)\,dx$.

8. Determine whether the integral $\int_0^{\infty} \frac{dx}{x^2+4}$ is convergent or divergent.

9. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} d\theta \ dr$.

10. Evaluate $\int_0^1 \int_0^2 \int_0^3 \left[x\,y^2\,z\right] dx\,dy\,dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. (8)

(ii) Using Cayley — Hamilton theorem find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1 x_2 + 2x_1 x_3 - 2x_2 x_3$ to canonical form through an orthogonal transformation. Also find its nature, rank, index and signature. (16)

12. (a) (i) If $x^2+y^2=25$, then find $\frac{dy}{dx}$ and also find an equation of the tangent line to the curve $x^2+y^2=25$ at the point $(3,4)$. (8)

(ii) If $f(x)=xe^x$ then find $f'(x)$. Also find the n-th derivative $f^n(x)$. (8)

Or

(b) (i) Differentiate the function $f(x) = \frac{\sec x}{1+\tan x}$. For what values of x, the graph of $f(x)$ has a horizontal tangent? (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$. (8)

13. (a) (i) If $u = \log[\tan x + \tan y + \tan z]$ then find the value of $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}$. (8)

(ii) Find the minimum value of $f(x, y) = x^2 + y^2 + 6x + 12$. (8)

Or

(b) (i) Expand $f(x, y) = e^x \sin y$ in terms of powers of "x" and "y" up to third degree terms by using Taylor's series. (8)

(ii) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (8)

14. (a) (i) Evaluate $\int \cos^n x \, dx$ by using integration by parts. (8)

(ii) Evaluate $\int \frac{dx}{\sqrt{3x - x^2 - 2}}$. (8)

Or

(b) (i) Evaluate $\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$ by using the method of partial fractions. (8)

(ii) Evaluate $\int \frac{2x + 3}{x^2 + x + 1} dx$. (8)

15. (a) (i) Evaluate $\iint [x y] \, dx \, dy$ where the region of integration is bounded by the lines x-axis, $x = 2a$ and the curve $x^2 = 4ay$. (8)

(ii) Change the order of the integration in $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} [x y] \, dy \, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\int_0^a \int_y^a \left[\frac{x}{x^2 + y^2} \right] dx \, dy$ by changing into polar coordinates. (8)

(ii) Evaluate $\int_0^{2a} \int_0^x \int_y^x [x \, y \, z] \, dz \, dy \, dx$. (8)

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